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Information measures of a deformed harmonic oscillator in a static electric field

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HIGHLIGHTS

- Harmonic oscillator under a static electric field in a space with modified metric.
- We calculate the Shannon entropy for the system.
- We calculate the Fisher information for the system.

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ABSTRACT

The Shannon entropy and the Fischer information are calculated for an harmonic oscillator in the presence of an applied electric field (ε) in a space with metrics given by $g_{xx}^{-1/2} = 1+\gamma x$. For that metric the harmonic oscillator can be mapped into a Morse potential in an Euclidean space. For $\varepsilon = 0$, the ground state energy decreases when γ increases. However, for certain values of ε the energy decrease can be canceled out. The dependence of the uncertainties, the entropy, and the information on the parameters γ and ε are shown.

The Fischer information and Shannon entropy are close related to the Heisenberg uncertainty principle which is one of the pillars of quantum mechanics. But even such pillar has been modified to take into account the quantization of space–time [1], string theory [2], and quantum gravity [3]. The main feature of these theories is to suggest a non-zero minimum uncertainty in the position from a Generalized Uncertainty Principle (GUP) [4–8].

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}\left[1+\alpha^2\left(\langle p\rangle^2+\Delta p^2\right)\right], \tag{1}$$

that was derived from the commutation relation $[x, p] = i\hbar(1+\alpha^2 p^2)$. For this particular commutation relation, the Shannon entropy for the one-dimensional harmonic oscillator was studied using the self-adjoint representation [9,10] and the validity of the Bialynicki-Birula–Mycielski inequality [11,12] in the GUP framework was verified.

In order to give a more symmetrical description of GUP a minimum momentum has been proposed to study deSitter black hole thermodynamics [13–16];

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}\left[1+\beta^2\left(\langle x\rangle^2+\Delta x^2\right)\right]. \tag{2}$$

Eq. (2) is derived from a commutation relation $[x, p] = i\hbar(1+\beta^2 x^2)$ giving the minimum momentum $\Delta p_{x min} = \hbar\beta$. The above commutation relation is not derived from first principles. Recently, Costa Filho et al. [17–20] considered a translation

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operator acting in a space with a diagonal metric, and showed that the momentum operator is modified leading to the commutation relation

$$[x, P_x] = i\hbar g_{xx}^{-1/2}. \tag{3}$$

The metric term in the above equation can be considered as a general function of position and can be Taylor expanded as $g_{xx}^{-1/2} = \sum_0^{\infty} a_n x^n$.

Here we calculate the Shannon entropy and Fisher information for the harmonic oscillator under the influence of a static electric field in a space with metrics defined by $g_{xx}^{-1/2} = 1 + \gamma x$. With this, the momentum is given by $P_x = (1 + \gamma x)p_x$. It is important to mention that, up to first order in position, the harmonic oscillator (HO) in this non-Euclidean space can be mapped into a Morse potential in an Euclidean space [18].

In order to maintain the hermiticity of x and P_x the inner product between two functions is defined as

$$\langle \phi|\psi\rangle \equiv \int \phi^*(x)\psi(x)\sqrt{g_{xx}}dx. \tag{4}$$

In this context, a particle in the vicinities of a point with coordinate x can be described by the ket $|x\rangle$ where $\hat{x}|x\rangle = x|x\rangle$. As the set $\{|x\rangle\}$ is complete, the identity operator can be written as

$$1 = \int \sqrt{g_{xx}}dx|x\rangle\langle x|, \tag{5}$$

and the scalar product in this metric for one dimension is given by $\langle x|x'\rangle = (1+\gamma x)\delta(x-x')$. It is important to mention that hermiticity is not a necessary condition to work in problems with minimum length [21].

The above-described representation is not adequate to calculate the Shannon entropy and Fisher information, since the wave functions of position in space and momentum are not connected by a Fourier transform. To overcome this problem we change the variables of the problem

$$\eta(x) \equiv \int \sqrt{g(x)}dx, \tag{6}$$

and in the particular case of $\sqrt{g(x)} = (1+\gamma x)^{-1}$ we have that $\eta = \ln(1+\gamma x)/\gamma$, and now

$$\int d\eta|\eta\rangle\langle\eta| = 1, \tag{7}$$

$$\langle\eta|\eta'\rangle = \delta(\eta-\eta') \tag{8}$$

$$P_x u_p(\eta) = p_x u_p(\eta), \tag{9}$$

where $u_p(\eta) = \langle\eta|p\rangle$ and can be expressed as

$$u_p(\eta) = \frac{1}{\sqrt{2\pi}}e^{ip_x\eta/\hbar}. \tag{10}$$

That allows us to write the wavefunction in momentum space as

$$\phi(p_x) = \frac{1}{\sqrt{2\pi}}\int \psi(\eta)e^{-ip_x\eta/\hbar}d\eta, \tag{11}$$

and taking the Fourier transform we arrive to the wavefunction in the position space

$$\psi(\eta) = \frac{1}{\sqrt{2\pi}}\int \phi(p_x)e^{ip_x\eta/\hbar}dp_x. \tag{12}$$

It is important to mention that, up to third order in the metric expansion $g_{xx}^{-1/2} = 1+\gamma x+\beta^2x^2$, Eq. (6) allows one to map any Hamiltonian in a space with metrics, given by $H(x, P_x) = P_x^2/2m + V(x)$, into the following Hamiltonian in the Euclidean space

$$H(\eta, p_x) = \frac{p_x^2}{2m} + V_{eff}(\eta). \tag{13}$$

To analyze the effects of the metric in the localization of a particle subject to an effective potential $V_{eff}(\eta)$, we calculate the information measures of localization called Fisher information [22] and Shannon entropy [23]. Fisher information is a quality of an efficient measurement procedure used for estimating ultimate quantum limits [24,25]. The Shannon entropy, introduced in 1948, was firstly employed to study fundamental limits on signal processing operations. Later on, it was used in Quantum Mechanics to determine the uncertainty related to the particle position, or in other words, with the degree of localization. There are several works reporting on the calculation of both the Fisher information and Shannon entropy for several time-independent and time-dependent quantum systems [26–32].