



q -triplet for Brazos River discharge: The edge of chaos?

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HIGHLIGHTS

- Brazos River discharge data exhibit q -triplet behavior.
- This is the first such finding in hydrological data.
- q -triplet values are intriguingly close to those of the Figenbaum point.

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ABSTRACT

We study the daily discharge data of Brazos River in Texas, USA, from 1900 to 2017, in terms of concepts drawn from the non-extensive statistics recently introduced by Tsallis. We find that the Brazos River discharge indeed follows non-extensive statistics regarding equilibrium, relaxation and sensitivity. Besides being the first such finding of a full-fledged q -triplet in hydrological data with possible future impact on water resources management, the fact that *all three* Tsallis q -triplet values are remarkably close to those of the logistic map at the onset of chaos opens up new questions towards a deeper understanding of the Brazos River dynamics, that may prove relevant for hydrological research in a more general sense.

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1. Introduction

Hydrologic measurements are fundamental for planning, design, operation, and management of water resources systems. While diverse statistical and theoretical methods have been developed over the past decades to address the nonlinearity and complexity of these data, such as chaos theory [1,2], multifractal analysis [3–6], information theory [7–9], and various entropy measures [10–14], their full understanding remains an open challenge.

Non-extensive statistics introduced recently by Tsallis [15] represents a strong candidate for a framework that may provide a considerable contribution to shedding new light on hydrologic phenomena, although to the best of our knowledge up to date there have been no applications of non-extensive statistics in hydrology. From a mathematical standpoint, Tsallis statistics generalizes Boltzmann Gibbs (BG) statistics by substituting exponentials with q -exponentials

$$\exp_q(x) = [1 + (1 - q)x]^{1/(1-q)},$$

and natural logarithms with q -logarithms

$$\ln_q(x) = \frac{x^{1-q} - 1}{1 - q},$$

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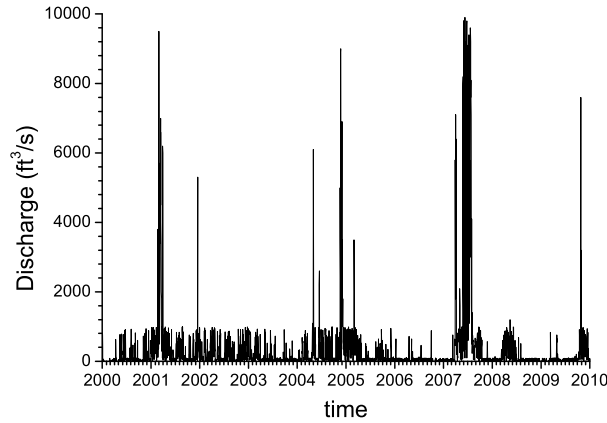


Fig. 1. A ten year window of discharge data for the Waco gauge station on Brazos River.

which both tend to their classical counterparts as $q \rightarrow 1$. The implications of this seemingly simple generalization to physical systems is enormous: (i) stationary probability distributions acquire long tails, (ii) stationary states turn out less sensitive to initial conditions, and (iii) relaxation towards equilibrium becomes slower (q -exponential, rather than exponential). The term “non-extensivity” here refers to the fact that entropy is not additive: entropy of a system is not a simple sum of its sub-system entropies. In terms of time series data analysis these three distinct features translate into: (i) distributions with long (power law) tails, (ii) wider multifractal spectrum, and (iii) q -exponential decay of correlations, respectively. The set of values of the non-extensivity parameter q stemming from each of these three analyses are denoted as “ q -triplet”. The q -triplet was found in natural phenomena such as ozone layer [16], solar plasma [17,18], El Niño Southern Oscillation [19], and geological faults [20], as well as in paradigmatic low-dimensional nonlinear dynamical systems, such as the logistic map near the edge of chaos [21], and the standard map [22].

In this work we analyze daily discharge data of Brazos River in Texas, USA, from 1900 to 2017, and we find that indeed the seemingly erratic intermittent discharge behavior may be well described by non-extensive statistics. Moreover, we find that *all three* q -triplet values match closely those of the logistic map for the parameter value that separates regions of deterministic and chaotic behavior — known as “the edge of chaos”. The deceiving simplicity of the logistic map, together with this analogy of the q -triplets seen as dynamic behavior quantifiers, encourages further research into the origins of this correspondence. More precisely, while chaos theory has been applied to hydrological data in numerous works over the last decades (see e.g. [2] and references therein), the current findings of a full-fledged q -triplet behavior of Brazos River discharge suggest that perhaps an intuitively simple (albeit theoretically intractable) mechanism may be responsible for this correspondence, promising a deeper insight into the phenomenon.

2. Streamflow data

Streamflow data for this study were obtained from the National Water Information System (Web Interface at <https://waterdata.usgs.gov/nwis>), for the Brazos River in Texas, with a drainage area of approximately 118,000 km², extending from eastern New Mexico to more than 1000 km southeast, to the Gulf of Mexico [23]. A total of $N = 42,758$ daily streamflow observations from 1/1/1900 to 1/26/2017 of the Waco stream gauge on the mainstream was selected for this study, as it exhibits the highest entropy (carrying most information) as compared with other gauge stations on this river [14]. The mean of 2377 ft³/s, standard deviation of 5841 ft³/s, zero minimum, and median of 740 ft³/s and maximum of 158,000 ft³/s reflect the extremely long tailed distribution of discharge values. The intermittent behavior is seen in Fig. 1 (only a ten year window is shown for better clarity), where no systematic periodicity can be observed.

Nevertheless, Fourier transform analysis displayed in Fig. 2 indicates a pronounced periodicity impact once, twice, and three times per year, and therefore for each calendar day i , for the period under study we find the mean μ_i and standard deviation σ_i over the years $j = 1900, \dots, 2016$, and we normalize the discharge data to find the discharge anomalies $z_{i,j} = (x_{i,j} - \mu_i) / \sigma_i$, displayed in Fig. 3.

Following [16], the normalized series z_n , $n = 1, \dots, N$, are used for correlation analysis, while for the stationary PDF and multifractal analysis we further consider anomaly increments $\Delta z_n = z_{n+1} - z_n$, displayed in Fig. 4.

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