Physica A 497 (2018) 246–257

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Effect of interaction strength on robustness of controlling edge dynamics in complex networks

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HIGHLIGHTS

- A framework is proposed to quantify the robustness of edge controllability.
- Robustness is determined jointly by interaction strength and degree distribution.
- A method is proposed to optimize the robustness of edge controllability.

ARTICLE INFO

Article history:
Received 28 September 2017
Received in revised form 28 November 2017
Available online 5 March 2018

ABSTRACT

Robustness plays a critical role in the controllability of complex networks to withstand failures and perturbations. Recent advances in the edge controllability show that the interaction strength among edges plays a more important role than network structure. Therefore, we focus on the effect of interaction strength on the robustness of edge controllability. Using three categories of all edges to quantify the robustness, we develop a universal framework to evaluate and analyze the robustness in complex networks with arbitrary structures and interaction strengths. Applying our framework to a large number of model and real-world networks, we find that the interaction strength is a dominant factor for the robustness in undirected networks. Meanwhile, the strongest robustness and the optimal edge controllability in undirected networks can be achieved simultaneously. Different from the case of undirected networks, the robustness in directed networks is determined jointly by the interaction strength and the network's degree distribution. Moreover, a stronger robustness is usually associated with a larger number of driver nodes required to maintain full control in directed networks. This prompts us to provide an optimization method by adjusting the interaction strength to optimize the robustness of edge controllability.

Complex networks composed of interacting dynamic units are common in many natural, social and technological systems [1–4]. One of the fundamental issues in modern network science is how to control complex network. Liu et al. [5] made a breakthrough by developing structural control theory and provided an efficient method based on maximum matching to determine the minimum number of driver nodes. Correspondingly, Nepusz et al. [6] introduced the switchboard dynamics (SBD) to describe the edge dynamics in complex networks. Much interest has been motivated in the research of controllability properties of complex networks [7–12].

As an important part of network science, the robustness [13–20] plays a key role in many natural, social and technological systems with complex topological features. Particularly, it is crucial for many infrastructure networks such as power

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https://doi.org/10.1016/j.physa.2018.01.012
0378-4371/

grids [16], Internet [17] and transportation networks [18]. Some evidences have demonstrated that such networks can be affected by intentional attacks or random failures emerging locally. To see the robustness of nodal controllability in complex networks under unavoidable node (edge) failures, Liu et al. [5] classified each node (edge) into three categories (critical, redundant and ordinary) according to the change in the number of driver nodes when the node (edge) is removed. They found that the content of cores and leaves in a network is the key factor which determines the proportion of three node (edge) categories. Similarly, Nepusz et al. [6] analyzed the robustness of edge controllability against edge failures, and found that the category of an edge can be determined by its local information, i.e., the in- and out-degrees of its end nodes. Following this work, we analyzed the robustness of edge controllability against node failures [21], and found that the proportion of three node categories is effected mainly by the network's degree distribution.

Most studies of the network controllability are performed based on the structural controllability, in which all interaction strengths are divided into zero and independent free parameters. In contrast, Yuan et al. [22] provided an exact controllability paradigm to determine the minimum number of driver nodes required to maintain full control. It can be applied to complex networks with arbitrary structures and interaction strengths, where the interaction strength is the link-weight among nodes. Subsequently, we expanded the SBD to the generalized switchboard dynamics (GSBD) [23] to characterize a dynamical process on edges of directed and undirected networks with arbitrary interaction strengths among edges. Many findings of controllability properties of the GSBD significantly differ from those of the nodal dynamics and the structural edge controllability. Specifically, the interaction strength plays a more important role in the edge controllability than network structure. Particularly, there exist upper and lower bounds of the number of driver nodes, which are determined by the interaction strength.

The significant role of the interaction strength in the edge controllability prompts us to explore its effect on the robustness. Following Liu et al. [5], we classify edges into three categories by the effect of removing them on the number of driver nodes. This classification leads to a quantitative analysis of the robustness of edge controllability. We prove that the category of an edge can be determined by the information of its end nodes, including the in- and out-degrees and the rank of general switching matrices. Based on the criterion of discerning edge category, a universal framework is proposed to evaluate and analyze the robustness in complex networks with arbitrary structures and interaction strengths. Simulation results and analytic calculations reveal a number of properties associated uniquely with the robustness of edge controllability. Firstly, we find that the interaction strength is a dominant factor for the robustness in undirected networks. Conversely, the robustness in directed networks is determined jointly by the interaction strength and the network's degree distribution. Secondly, we find that the strongest robustness and the optimal edge controllability in undirected networks can be achieved simultaneously. Conversely, a trade-off exists between the robustness and the edge controllability in directed networks. That is, a stronger (weaker) robustness is usually associated with a larger (smaller) number of driver nodes required to maintain full control. Finally, we provide an optimization method based on adjusting the interaction strength to optimize the robustness. The simulation results of model and real-world networks show the feasibility and effectiveness of the optimization method.

1. Results

1.1. General switchboard dynamics

The GSBD [23] provides a general characterization of dynamics occurring on edges of a network $G(V, E)$. Let $\mathbf{y}_v^-$ and $\mathbf{y}_v^+$ denote the state vectors comprised of the states of the incoming and outgoing edges of node v, respectively. The state vector $\mathbf{y}_v^+$ can be influenced by the state vector $\mathbf{y}_v^-$ and the input vector $\mathbf{u}_v$. So the equation governing the edge dynamics is

$$\dot{\mathbf{y}}_v^+ = S_v \mathbf{y}_v^- + \sigma_v \mathbf{u}_v, \tag{1}$$

where $S_v \in \mathbb{R}^{k_v^+ \times k_v^-}$ is the general switching matrix with row numbers equaling the out-degree k_v^+ and column numbers equaling the in-degree k_v^- of node v. The element in S_v captures the interaction strength among edges. In the structural controllability [6], S_v must be a structural matrix, in which all nonzero elements are independent free parameters. Instead, the GSBD releases the restriction of S_v, in which the elements could be arbitrary fixed values. σ_v is 1 if node v is a driver node and 0 otherwise.

Let $\mathbf{x} = (x_1, x_2, \dots, x_M)^T$ denote the state vector comprised of the state of each edge. A correspondence between the GSBD and the linear time-invariant dynamical system can be established by rewriting Eq. (1) in terms of x_i, yielding

$$\dot{\mathbf{x}} = W\mathbf{x} + H\mathbf{u}, \tag{2}$$

where $W \in \mathbb{R}^{M\times M}$ is the transpose matrix of the adjacency matrix of the line graph $L(G)$ of G, in which w_{ij} is nonzero if and only if the head of edge j is the tail of edge i. $H \in \mathbb{R}^{M\times M}$ is a diagonal matrix where the ith diagonal element is σ_v if node v is the tail of edge i.

For the edge dynamics in undirected networks, the GSBD can be defined by splitting each undirected edge into two directed edges with opposite directions. Each undirected edge is denoted by a couple of state variables (x_i, x_i') corresponding to its two directed edges. As a result, the state vector of the dynamical process occurring on undirected edges is $\mathbf{x} =$