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Coexistence of several currencies in presence of increasing returns to adoption

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HIGHLIGHTS

- A simple model of competition between different currencies.
- Takes into account the specificity of local situations through the introduction of a network of exogenously fixed bilateral links.
- Mathematical proof of the possible existence of multiple currencies at equilibrium.

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ABSTRACT

We present a simplistic model of the competition between different currencies. Each individual is free to choose the currency that minimizes his transaction costs, which arise whenever his exchanging relations have chosen a different currency. We show that competition between currencies does not necessarily converge to the emergence of a single currency. For large systems, we prove that two distinct communities using different currencies in the initial state will remain forever in this fractionalized state.

1. Introduction

In general, a currency is useful – and therefore sought by an individual – only insofar as it can be used to buy goods. This implies that the currency is widely accepted as payment by her suppliers. In a world where several currencies exist, the attractiveness of a currency for an individual can be measured by its greater or less acceptance in the group of individuals with whom it is used to exchange goods. It follows that the more a currency is accepted, the greater its attractiveness becomes. From this point of view, money is akin to language [1]. The assumption that in cross-currency competition, increasing returns to adoption play a fundamental role has been present in economic theory since the famous article by Carl Menger [2]. The same idea has been examined by various authors in the Economics literature [3–10] as well as in Physics journals [11–13]. Recent models have pushed further Menger's basic idea either by trying to introduce social aspects of money [8], or providing a unified framework able to explain at the same time the emergence of a single currency and some other economic phenomena. Yasutomi's [11] model, further refined by [12,13], can also describe the collapse of a currency, while Donangelo and Sneppen's links the emergence of money to its anomalous fluctuation in value [14]. Duffy and Ochs [7] tested some predictions of Kiyotaki and Wright model [4] in laboratory experiments. Another stream of literature connects the emergence of money to the more general question of the emergence of social norms using game theory [5,7,6,10].

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In this article, we propose a model that differs from all these approaches by two characteristics : its exchange mechanism is simpler and it takes into account the specificity of local situations through the introduction of a network of exogenously fixed bilateral links, for example because of constraints originating in the social division of labour. On the first point, most previous work assume some more realistic exchange mechanism. For example, models inspired by Yasutomi's model [11–13] take, as elementary interaction between two agents, the transaction, which "consists of several steps including search of the co-trader, exchange of particular goods, change of the agent's buying preferences and finally the production and consumption phase". On the second point, unlike most models, we do not assume a completely connected interaction network, which leads to simpler analytical treatments but obscures the local aspects of economic transactions. We demonstrate that, under such conditions, competition between currencies does not necessarily converge to the emergence of a single currency. Even if an equilibrium with a single currency remains possible, the most frequent stable configuration is the division of the trading space between different currencies. For large systems, we prove that two distinct communities using different currencies in the initial state will remain forever in this fractionalized state.

2. General framework

We consider an economy composed of N agents, numbered $i = 1, \ldots, N$, each starting with its own currency s_i. A currency will be referred to as an integer in $[1, N]$, and we assume that each agent begins with its own currency $s_i = i$. The agents are disposed on the vertices of a random graph [15], whose edges represent commercial links between the agents. After choosing a density of links $p \in [0, 1]$, for each pair of agents (i, j), we create a link between i and j with probability p, or let the agents disconnected with probability $(1 - p)$.

The interaction dynamics is set in the following way. Each time step, we choose an agent at random. First, this agent is allowed to change the currency it uses. Then, it trades with all its neighbours, *i*.e. with all the agents he shares a link with. The profit an agent gets is defined according to the following idea: if two agents share the same currency, their trading business is made easier and no cost has to be paid; conversely, if they use different currencies, they must trade through the help of some "moneychanger" who gets a commission for its work: we will then consider that those agents have to afford some fixed cost (which we will take as a unit cost) in order to complete their trade. As there is no other constraint, we translate the profit of each particular trade to the origin so that a successful trade is worth 0 and a trade which has to resort to a moneychanger is worth -1.

We define a simple utility function $U_i(t)$ for an agent i at period t, as the opposite of the sum of all trading costs an agent has to pay when realizing its trades at period t:

$$U_i = -\sum_{j \in \mathcal{V}(i)} (1 - \delta_{s_i}^{s_j}) = -Card\{j \in \mathcal{V}(i) | s_j \neq s_i\}$$

where $\mathcal{V}(i)$ is the set of the neighbours of the agent i in the graph and $\delta_{s_i}^{s_j}$ is the Kronecker symbol, ie $\delta_{s_i}^{s_j} = \begin{cases} 1 \text{ if } s_i = s_j \\ 0 \text{ if } s_i \neq s_j \end{cases}$.

We also define a social utility function for the economy as a whole, as the sum of the utilities of all agents:

$$\mathcal{U} = \sum_{i=1}^{N} U_i = -\frac{1}{2} \sum_{i,j \text{ neighbours}} (1 - \delta_{s_i}^{s_j})$$

where the factor $\frac{1}{2}$ accounts for the fact that each pair of neighbours is counted twice in the sum.

As we assume that agents are fully rational and maximize their own utility function, the rule for currency adoption is the simplest mimetic one: an agent adopts the most common currency among its neighbours; if several currencies are used by the same maximal number of neighbours, two cases appear: either the agent already uses one of them, and keeps using it by default, or she was using another one, in which case she picks at random one of the most popular currencies among her neighbours. Agents do not take into account neither the anticipated cost of future trades (for $t' > t$) nor the influence their choice could have on other agents.

From the evolution rule we have defined, we can infer that social utility can only increase with time; more, it increases strictly when an agent changes its currency. Indeed, if agent i switches from currency s to currency s' at time t, social utility increases with

$$\begin{aligned} \Delta\mathcal{U} &= \Delta U_i + \sum_{j \in \mathcal{V}(i)} \Delta U_j \\ &= \sum_{j \in \mathcal{V}(i)} \delta_{s'}^{s_j} - \delta_{s}^{s_j} + \sum_{j \in \mathcal{V}(i)} \delta_{s_j}^{s'} - \delta_{s_j}^{s} \\ &= 2\left(Card\{j \in \mathcal{V}(i) | s_j = s'\} - Card\{j \in \mathcal{V}(i) | s_j = s\} \right) \end{aligned}$$

where we write $s_j = s_j(t-1) = s_j(t)$ for all $j \in \mathcal{V}(i)$. By definition, agent i switches from s to s' at time t if and only if

$$Card\{j \in \mathcal{V}(i) | s_j = s'\} > Card\{j \in \mathcal{V}(i) | s_j = s\},$$