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Performance analysis for minimally nonlinear irreversible refrigerators at finite cooling power

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HIGHLIGHTS

- The minimally nonlinear model is adopted to analyze the performance of refrigerators .
- COP and its bounds at given cooling power for different operating regions are deduced.
- COP bounds under the χ figure of merit at finite cooling power are calculated.
- Relative gain in COP in different operating regions is presented.

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ABSTRACT

The coefficient of performance (COP) for general refrigerators at finite cooling power have been systematically researched through the minimally nonlinear irreversible model, and its lower and upper bounds in different operating regions have been proposed. Under the tight coupling conditions, we have calculated the universal COP bounds under the χ figure of merit in different operating regions. When the refrigerator operates in the region with lower external flux, we obtained the general bounds $(0 < \varepsilon < (\sqrt{9+8\varepsilon_C}-3)/2)$ under the χ figure of merit. We have also calculated the universal bounds for maximum gain in COP under different operating regions to give a further insight into the COP gain with the cooling power away from the maximum one. When the refrigerator operates in the region located between maximum cooling power and maximum COP with lower external flux, the upper bound for COP and the lower bound for relative gain in COP present large values, compared to a relative small loss from the maximum cooling power. If the cooling power is the main objective, it is desirable to operate the refrigerator at a slightly lower cooling power than at the maximum one, where a small loss in the cooling power induces a much larger COP enhancement.

1. Introduction

Refrigerators are essential thermodynamic devices in our everyday life, which are widely used for cooling and refrigeration systems. Driven by the external power, a refrigerator absorbs heat from the cold reservoir with temperature T_c, and release it into the hot reservoir with temperature T_h. Based on the second law of thermodynamics, the coefficient of performance (COP) for a traditional refrigerator is bounded by the Carnot limit, $\varepsilon_C = T_c/(T_h - T_c)$ [1], which is usually achieved with infinite cycle duration, meanwhile the cooling power vanishes, which is unrealistic for actual applications

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although the attainability of Carnot efficiency at a non-zero power in some special cases has been reported recently [2–7]. Pioneered by Curzon and Ahlborn [8], who considered finite cycle duration in studying practical heat engines, finite time thermodynamics provides an alternative way to study and optimize heat devices to fulfill actual demands. And many models have been developed to describe the operation characteristics of actual thermodynamic cycles, such as the endoreversible model [8–10], low-dissipation model [11–13], irreversible models based on the Onsager relation [14–17], quantum models [18–22], and Brownian models [23–25]. If the protocol driving the engine is optimized to reach maximum amount of work during isothermal branches of the cycle, the stochastic heat engine can be tuned to fit the low-dissipation model [26–28], which is only a special case of the minimally nonlinear irreversible model [17].

The universal optimization criterion for analyzing the performance of refrigerators is widely discussed. Many criteria have been proposed to obtain the general bounds for the coefficient of performance to describe the actual operating conditions of the refrigerators, such as the maximum cooling power criterion [29], the maximum COP criterion [30], the Ω criterion [31], the χ criterion [12] and the ecological criterion [32]. And useful results have been provided. Under the maximum cooling power condition, the COP lies in the range $0 < \varepsilon < \varepsilon_C/(2+\varepsilon_C)$ [33]. However it is not in accord with the operating performance of actual refrigerators. Further study reveals that the real-life working conditions do not correspond to a maximum cooling power but rather to a trade-off between maximum cooling power and maximum COP [29,34]. The Ω criterion represents a compromise between the energy benefit and losses [10,17], and the χ criterion (the product of the COP and the cooling power) indicates a trade-off between the maximum cooling power and maximum COP [35]. Through studying the low dissipation refrigerator, Wang et al. [36] proposed that the COP at maximum χ was bounded between 0 and $(\sqrt{9+8\varepsilon_C}-3)/2$, which is also obtained through the minimally nonlinear irreversible refrigerators [16]. And the CA coefficient of performance $\varepsilon_{CA}=\sqrt{\varepsilon_C+1}-1$ is recovered under the conditions of symmetric dissipations. By investigating general endoreversible refrigerators with non-isothermal processes, the CA coefficient of performance is also deduced under maximum χ criterion [9]. In addition, Long and Liu [22] analyzed the performance of a quantum Otto refrigerator coupled to a squeezed cold reservoir using the χ figure of merit. They claimed that the COP under the maximum χ figure of merit is of the Curzon–Ahlborn (CA) style that cannot surpass the actual Carnot COP.

However, the above χ figure of merit is an ad-hoc criterion, the general universal optimization criterion should be further investigated. There probably does not exist a general universal optimization criterion, but one always take a criterion which fits his specific application. This means that the knowledge of maximum coefficient of performance at any finite cooling power is even more important, which could offer special guidance to operate actual refrigerators under the desired conditions. Motivated by the study of the efficiency at an arbitrary power output [28,37–41], and as a counterpart of our previous paper on studying the performance of the minimally nonlinear heat engines at finite power [42]. In present paper, we first introduce the model of minimally nonlinear refrigerators, and systemically discuss the coefficient of performance and relative gain in coefficient of performance at any arbitrary cooling power for minimally nonlinear refrigerators, respectively. Under the tight coupling conditions, the COP bounds at any finite cooling power are proposed and the COP bounds in different operating regions under the χ figure of merit are deduced. Furthermore, under the non-tight coupling conditions, general COP bounds and the bounds for the relative gain in COP at any arbitrary cooling power are also proposed, and some extended discussions are presented to give a deep insight into the general COP for finite cooling power. Finally, some important conclusions are drawn.

2. Minimally nonlinear irreversible refrigerators

For refrigerators, the cooling load ($\dot{Q}_c$) is absorbed from the cold reservoir (T_c), and a certain amount of heat ($\dot{Q}_h$) is evacuated to the hot reservoir (T_h) at the end of a cycle. After a cycle, the working fluid in the refrigerators returns to its initial state; therefore, its entropy change per cycle is zero. The total entropy production rate $\dot{\sigma}$ of the refrigerator can be written as

$$\dot{\sigma}=\frac{\dot{Q}_h}{T_h}-\frac{\dot{Q}_c}{T_c}=\frac{\dot{W}}{T_h}+\dot{Q}_c\left(\frac{1}{T_h}-\frac{1}{T_c}\right) \quad (1)$$

where the dot denotes the quantity per unit time for simultaneous refrigerators (working in a time-independent steady state), or the quantity divided by the cycle time duration for sequential refrigerators (working in a time-periodic steady state). Eq. (1) suggests considering a driving force $X_1=F/T_c$ associated with the external force F performing work with thermodynamically conjugate variable x and a flux $J_1=\dot{x}$, so that the input power can be rewritten as $\dot{W}=F\dot{x}=J_1X_1T_h$; the other thermodynamic force and its conjugate flux can also be defined as $X_2=1/T_h-1/T_c$ and $J_2=\dot{Q}_c$, respectively. In linear irreversible refrigerators, the relations of the thermodynamic fluxes and forces are governed by linear relations [43]. By adding a nonlinear term to the linear relations to consider the power dissipation into the heat reservoirs due to the fraction loss, the minimally nonlinear irreversible model was proposed, and extended Onsager relations are adopted to illustrate the refrigerators [16,44,45]:

$$J_1=L_{11}X_1+L_{12}X_2 \quad (2)$$

$$J_2=L_{21}X_1+L_{22}X_2-\gamma_cJ_1^2 \quad (3)$$