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Dynamical and topological aspects of consensus formation in complex networks

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HIGHLIGHTS

- We present a model for consensus formation.
- We study the effect of different network topologies on the emergence of consensus.
- We conclude that heterogeneous topologies inhibit the formation of consensus.

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ABSTRACT

The present work analyzes a particular scenario of consensus formation, where the individuals navigate across an underlying network defining the topology of the walks. The consensus, associated to a given opinion coded as a simple message, is generated by interactions during the agent's walk and manifest itself in the collapse of the various opinions into a single one. We analyze how the topology of the underlying networks and the rules of interaction between the agents promote or inhibit the emergence of this consensus. We find that non-linear interaction rules are required to form consensus and that consensus is more easily achieved in networks whose degree distribution is narrower.

1. Introduction

The role of complex networks as a mathematical tool to formalize the underlying topology in several propagating phenomena of social extraction has been well established in the last years. When looking for models of the spread of diseases or the propagation of information, rumors and ideas, complex networks provide a plethora of alternative topologies that serve to mimic the complex weave of the interpersonal relationships [1–4]. At a more abstract level, diffusion in general is one of the fundamental processes taking place in networks [5–7]. Diffusive propagation on a network generates the opportunity that agents get in touch and interchange ideas and information. Given the appropriate rules for interchange and network topology this could lead to the appearance of consensus among the opinions of the different agents.

Consensus formation is a widely studied phenomenon in social sciences. One of the main goals is to understand the emergence of consensus in a system involving a number of interacting agents. In [8] the authors study a model in which each agent can communicate with local neighbors and analyze the emergence of consensus. They find that increasing communication between agents who have common friends will prolong the time needed for the system to reach a consensus state. In [9,10] the authors study adaptive networks and show that the network topology fosters cluster formation by

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enhancing communication between agents of similar opinion. They also find that the rewiring process can lead to the elimination of interactions between agents with different opinions, accelerating the convergence to a consensus state but breaking the network into non-interacting groups. Another example worth mentioning is [11], where by means of the analysis of the Sznajd Model in one dimension they show that the transition rates towards a given opinion are directly proportional to the frequency of the respective opinion of the second-nearest neighbors.

This consensus dynamics has been also studied analytically, for instance, in [12–14], where it is proposed that each agent alters its opinion according to some weighted average of the rest of system. It is found that if all the recurrent states of the Markov chain communicate with each other and are aperiodic, then a consensus is always reached. In most of the existing models the update of opinions takes place via a linear mechanism [15–17]. Moreover, they do not usually analyze the influence of the network topology. Although see, for instance, [8], where a complex network appears as a substrate of a set of agents with linear dynamics and [18], where the topology of the network is altered by the interactions between the agents. The structure of the complex networks has been extensively studied from the point of view of transfer of information. A lot of emphasis has been put on the influence of topology, studying for instance whether scale-free networks are more efficient than regular ones [19]. In this context it has been shown, for instance, that it is very important whether the network consists of homogeneous nodes or it has a structure of routers and peripheral nodes. Another important aspect is the clustering of the nodes, because the presence or absence of loops could affect information transfer [19]. For instance, in networks with modular structures, it has been shown that the velocity of the information propagation depends non linearly on the number of modules. A piece of information will propagate faster for networks having either a small number or a large number of modules [20].

A set of interacting agents on a network can give rise to a dynamically changing local environments where the process of interchange of opinions takes place. In this paper we focus on three aspects: (1) How the local interaction rules control the convergence to consensus, in particular we analyze both linear and non-linear rules. (2) What is the influence of the dynamics of the agents in the network. We first consider the propagation of the information by considering a naïve strategy for neighboring node selection. If the target node is not among the neighbors, a neighboring node is selected at random for the trajectory of the agent. Then we consider a preferential choice strategy, where the agents are more likely to move to more connected nodes. (3) What is the effect of the different parameters of the network topology, such as clustering or assortativity. In the following sections we present the model in more details and a description of our results.

2. Network topologies

Throughout this work we use several families of networks with different topologies and algorithmic constructions, though always containing the same number of nodes and links.

Regular small world networks:. We consider first regular networks with a tuned degree of disorder, and consequently different degrees of clustering and mean path length. We recall that the clustering coefficient measures the tendency of the nodes to cluster together and can be locally characterized by the fraction of existent links between nodes in a given node neighborhood to the number of links that could possibly exist between them [21]. The mean path length is the averaged path length between all the nodes in the network. The path length is defined as the minimum number of links needed to navigate from one initial node to the final one.

Regular Small World networks are built using a modified algorithm based on the originally proposed in [21] to constrain the resulting networks to a subfamily with a delta shaped degree distribution. We call this family of networks the k-Small World Networks (k-SWN) [22], where $2k$ indicates the degree of the nodes. The building procedure starts with an ordered regular network whose order is broken by exchanging the nodes attached to the ends of two links in a sequential way. Starting, for example, from an ordered ring network, each link is subject to the possibility of exchanging one of its adjacent nodes with another randomly chosen link with probability p_d. If the exchange is accepted, we switch the partners in order to get two new pairs of coupled nodes. Double links are always avoided, thus if there is no way to avoid a double link with the present selection of nodes, a new choice is done. In this way all the nodes preserve their degree while the process of reconnection assures the introduction of a certain degree of disorder. It must be stressed that the dependence of the clustering coefficient C and path length L on the disorder parameter is qualitatively similar to the one observed in Small World Networks [21]. In this way we can evaluate the effects of clustering and path length independently of the degree distribution.

Small world networks:. To include the possible effects of the degree distribution on the analyzed dynamics we consider then, the usual algorithm described in [21], where only one link is rewired at a time, maintaining the attachment to one of the adjacent nodes and randomly connecting the other extreme. These networks not only possess different degrees of clustering and mean distance but also different binomial degree distributions, linked to the disorder parameter p_d, that measures the probability of changing the extremes of each link. The degree distribution of these networks changes from deltiform to binomial at the moment of introducing the slightest disorder. The associated binomial distribution is given by [23]

$$P(h) = \sum_{n=0}^{\min[h-k,k]} \binom{k}{n} (1-p_d)^n p_d^{k-n} \frac{(kp_d)^{h-k-n}}{(h-k-n)!} \exp(-kp_d), \tag{1}$$