Physica A 495 (2018) 172–179

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Preferential attachment in evolutionary earthquake networks

Soghra Rezaei *, Hanieh Moghaddasi, Amir Hossein Darooneh

Department of Physics, University of Zanjan, P.O. Box 45196-313, Zanjan, Iran

HIGHLIGHTS

- We have examined the preferential attachment for earthquakes network and showed that it is present in this case.
- The results reveal that the attachment rate has a linear relationship with node degree.
- We distinguished seismic passive points using their preferential attachment values.

ARTICLE INFO

Article history:
Received 8 August 2017
Received in revised form 10 October 2017
Available online 20 December 2017

Keywords:
Earthquakes network
Preferential attachment
Fault activity

ABSTRACT

Earthquakes as spatio-temporal complex systems have been recently studied using complex network theory. Seismic networks are dynamical networks due to addition of new seismic events over time leading to establishing new nodes and links to the network.

Here we have constructed Iran and Italy seismic networks based on Hybrid Model and testified the preferential attachment hypothesis for the connection of new nodes which states that it is more probable for newly added nodes to join the highly connected nodes comparing to the less connected ones. We showed that the preferential attachment is present in the case of earthquakes network and the attachment rate has a linear relationship with node degree. We have also found the seismic passive points, the most probable points to be influenced by other seismic places, using their preferential attachment values.

1. Introduction

Network theory is widely used to describe different features of complex systems in various fields [1]. To map these systems to a graph, nodes and links of the graph should be specified. The system members are often considered as the nodes and the interaction between them are defined as the links. Real world networks are evolving dynamical systems instead of static graphs mainly due to interactions with their environment; some nodes may be added or removed over time and subsequently their links will be changed such as dynamics of cells metabolic networks, social networks, World Wide Web and scientific collaborations [2–4].

Two common characteristics of evolving networks are their growth and preferential attachment [5]. The network growth means that new nodes and links between the nodes are added to the network. Preferential attachment hypothesis is proposed for connection of new nodes stating that new nodes tend to join highly connected nodes rather than the less connected ones which implies that the connectivity of the nodes with higher degrees increases much rapidly than the nodes with less degrees just as the rich get richer. Therefore the probability of connecting a new node to an existing node i with degree k_i depends on k_i:

$$\Pi(k_i) = \frac{k_i^\alpha}{\sum_j k_j^\alpha} = C(t)k_i^\alpha \tag{1}$$

---

* Corresponding author.
E-mail address: s.rezaee@znu.ac.ir (S. Rezaei).

https://doi.org/10.1016/j.physa.2017.12.063
0378-4371/

where $C(t)$ is the normalization constant and α is a positive exponent. If $\alpha = 1$ then preferential attachment is linear, the network is scale free and its degree distribution is power law [6]. For $\alpha < 1$, the preferential attachment is sub-linear in which the new nodes tendency to connect to the nodes with large degrees is less than the linear preferential attachment and the degree distribution is stretched exponential. For $\alpha > 1$, a single site connects to almost all other nodes and the preferential attachment is called super-linear [7].

Earthquakes are known as spatio-temporal complex phenomena due to complicated yet undiscovered dynamics of the earth crust [8]. Seismic complex systems have been recently described by utilizing complex network theory. One of the greatest advantages of using complex network theory for earthquakes is that the complete detailed profile of the system is not required. Only by obtaining the information about the location, time and magnitude of the seismic events, it will be possible to apply the theory for extracting diverse features of the seismic phenomenon.

The first step for constructing the earthquake network is to define what the nodes and their links are. Each seismic event can be regarded as a node and it is connected to other nodes if they are severely correlated. Baiesi et al. introduced a measure for the strength of the correlation between earthquakes based on time difference, spatial distance and magnitude of the first event [9]. Based on Telesca–Lovallo model, the nodes are linked together if they satisfy the visibility condition [10].

In another approach for defining nodes, the geographical region can be divided into identical square cells without overlapping. Any cell containing at least one earthquake location is regarded as a node. Abe and Suzuki proposed that two nodes are linked together if they are locations of two consecutive events. Different features of many networks in diverse fields are studied using this model [11–21]. Rezaei et al. combined Abe–Suzuki and Telesca–Lovallo models to construct the seismic network in which nodes are defined based on Abe–Suzuki model and they are linked using the visibility condition [22]. They showed that Gutenberg–Richter and Omori laws, two popular seismic laws, can be retrieved from their earthquake network.

Clearly earthquake network evolves over time by the addition of new seismic events leading to new nodes and links. According to preferential attachment hypothesis, connection of newly occurred seismic events to highly connected nodes is more probable than less connected ones [4]. Here we study the preferential attachment for dynamical earthquake networks. We first elaborate measuring preferential attachment in method section then the outcome for Italy and Iran seismic networks are demonstrated in the result section. Finally we summarize and discuss more about the results.

2. Method

In evolving networks, new nodes and links are added to the network over time. Connection of new nodes to the existing ones can be described by preferential attachment. To study preferential attachment for these kinds of networks, degree changes of existing nodes after the addition of the new nodes should be obtained. Then $\Pi(k)$ will be gained by plotting degree changes of old nodes versus their degrees according to Eq. (1). It is noticeable that $C(t)$ depends on time that new nodes attached the network which can lead to undesired biases in the calculations. In order to prevent the effects of such biases, we assume that new nodes join the network in quite short time intervals.

To calculate $\Pi(k)$ for a given network, all existing nodes at time T_0 are called "T_0 nodes", then "T_1 nodes" would be the nodes added to the network in time $[T_1, T_1 + \Delta T]$ where $\Delta T \ll T_1$ and $T_0 < T_1$. Degree changes of T_0 nodes after the time interval ΔT and addition of T_1 nodes should be calculated. So $\Pi(k, T_0, T_1)$ can be obtained through plotting Δk versus k. If $\Pi(k)$ does not depend on k, preferential attachment is not present [23].

Since the considered networks are large and ΔT must be chosen small enough as discussed earlier, several fluctuations appear in calculated $\Pi(k)$. Hence, it is more appropriate to deal with cumulative distribution function:

$$\kappa(k) = \int_0^k \Pi(k)dk \tag{2}$$

If $\Pi(k)$ depends on k as shown in Eq. (1), $\kappa(k)$ will be as follows:

$$\kappa(k) \propto k^{\alpha+1}. \tag{3}$$

If α is close to one, preferential attachment is present in the dynamical network and the degree distribution function of the network is scale free as explained in the previous section.

We have constructed Iran and Italy earthquake networks using Rezaei et al. model in which the Abe–Suzuki and Telesca–Lovallo models are mixed to define the nodes and links, respectively. In this model, time series of all seismic events $a, \ldots, c, \ldots, b$ are described by their locations, happening times and magnitudes as $(r_a, t_a, m_a), \ldots, (r_c, t_c, m_c), \ldots, (r_b, t_b, m_b)$. The geographical area is divided into identical small square cells. Any cell that includes at least one seismic event location is regarded as a node then two nodes will be linked if they satisfy the visibility condition; One can plot the earthquakes sequence as a bar graph in which x-axis is the time and the height of each bar is the corresponding event magnitude. Regarding the visibility condition, a hypothetical line drawn between two given events should not be cut by any other bar therein. In other words, the event a happening at t_a with magnitude m_a will be connected to event b occurring at t_b and magnitude m_b if the following inequality holds true for any event located between them called as c.

$$m_c < m_b + (m_a - m_b)\frac{t_b - t_c}{t_b - t_a} \tag{4}$$