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Stochastic resonance in multi-stable coupled systems driven by two driving signals

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HIGHLIGHTS

- The analytic expression for the amplitude of the response is obtained.
- The SR appears in the subsystem with weaker signal amplitude or even without signal with the help of coupling.
- The stochastic multi-resonance phenomenon is observed in the subsystem driven by low frequency signal.
- An effective scheme for phase suppressing SR is proposed in the coupled systems.

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ABSTRACT

The stochastic resonance (SR) in multi-stable coupled systems subjected to Gaussian white noises and two different driving signals is investigated in this paper. Using the adiabatic approximation and the perturbation method, the coupled systems with four-well potential are transformed into the master equations and the amplitude of the response is obtained. The signal-to-noise ratio (SNR) is calculated numerically to demonstrate the occurrence of SR. For the case of two driving signals with different amplitudes, the interwell resonance between two wells S_1 and S_3 emerges for strong coupling. The SR can appear in the subsystem with weaker signal amplitude or even without driving signal with the help of coupling. For the case of two driving signals with different frequencies, the effects of SR in two subsystems driven by high and low frequency signals are both weakened with an increase in coupling strength. The stochastic multi-resonance phenomenon is observed in the subsystem subjected to the low frequency signal. Moreover, an effective scheme for phase suppressing SR is proposed by using a relative phase between two driving signals.

1. Introduction

In recent years, the phenomenon of stochastic resonance (SR) has been extensively investigated and applied in various scientific areas [1–13], such as chemical reactions [1], energy harvesting [5], synthetic gene networks [8,9] and so on. However, the majority of works on SR considered bi-stable and mono-stable systems, only a few publications dealt with SR in multi-stable systems [14–19]. The dynamics of a Brownian particle in multi-stable systems is an important problem with extensive applications in many fields. For example, Arathi et al. [16] studied the influences of the depth of the wells and multi-fractal analysis on SR in a triple-well system. The SR phenomenon was found in an under-damped periodic potential driven by colored noise in Ref. [17], and the resonance region becomes wide as the correlation time of colored noise

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increases. In particular, the input periodic signal is of great significance to the study of SR in multi-stable systems. Li et al. [18] demonstrated that the SR in tri-stable model is more pronounced than that in mono-stable or bi-stable model by controlling the parameter values of potential function and input periodic signal. Reenbohn et al. [19] found that the SR and ratchet effect can occur in an under-damped periodic system when the appropriate parameter values of the external bi-harmonic signal exist. The above-mentioned papers only considered the effects of the single driving signal on SR. In practice, two different driving signals can play an important role in different fields, such as communication [20], lasers [21], and neuroscience [22]. Besides, the effects of different driving signals on SR and vibrational resonance were also shown in Refs. [23–26]. However, no attention has been paid to the influences of the two different driving signals on SR in multi-stable systems.

One of the research topics aimed at SR is the adding of system dimension, which would induce significant effects on both SR and its related dynamical phenomena. As expected, SR has been widely involved in multi-dimensional coupled systems [27–34]. For example, Kim et al. [27] studied the SR in N globally coupled oscillators with time delay. Yang et al. [28] explored the SR with synchronization in the system of N harmonically coupled particles with a randomly switching potential. Xu et al. [29,30] analyzed the influences of Lévy noise on the switch and SR in genetic toggle model. Both Neiman et al. [31] and Kenfack et al. [32] found that an optimal coupling strength can maximize the SNR in two coupled bi-stable systems. In fact, the research of SR in multi-dimensional systems with multi-stable potential would become more difficult due to the complexity of potential and the adding of system dimension. Thus, only a few publications have dealt with SR in two coupled systems with multi-stable potential. Nicolis [33] obtained the expressions of output response in two multi-stable coupled systems, and Gandhimathi et al. [34] found the occurrence of SR in two coupled over-damped oscillators with a four-well potential. Especially, the coupling parameter plays a crucial role in the resonance behaviors.

The aim of this study is to show a theoretical and numerical research on SR in multi-stable coupled systems driven by two different driving signals. The paper is organized as follows: in Section 2, a description of multi-stable coupled systems subjected to two driving signals and Gaussian white noises is given. In Section 3, the expression of the response amplitude is derived to characterize SR. Section 4 presents the SR for two driving signals with different amplitudes, frequencies or phases by using SNR. Finally, some conclusions are drawn in Section 5.

2. Model

Consider two coupled over-damped elements with noises and driving signals, the outputs of which are described by the following Langevin equations:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -\dfrac{\partial V(x,y)}{\partial x} + c(y-x) + F_x + \xi_x(t), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{\partial V(x,y)}{\partial y} + c(x-y) + F_y + \xi_y(t), \end{cases} \tag{1}$$

where c is the coupling strength, and $F_i = A_i \cos(\omega_i t + \varphi_i)(i = x, y)$ denote the external periodic signals. Here, A_i, ω_i, and φ_i are the amplitude, frequency, and phase of F_i, respectively. $\lambda = A_y/A_x$ is the amplitude ratio between two driving signals, $\eta = \omega_y/\omega_x$ is the driving frequency ratio, and $\Delta\varphi = \varphi_y - \varphi_x$ is the relative phase. The terms $\xi_i\ (t)$ are independent Gaussian white noises with zero-mean and Dirac δ correlation functions:

$$\left\langle \xi_i(t)\xi_j(t') \right\rangle = 2D\delta_{ij}\delta(t-t'), \quad i, j = x, y, \tag{2}$$

where D is the noise intensity.

The four-well potential $V(x, y)$, which has been employed in two coupled systems for the study of vibrational resonance [25,34], competition between two species [35], and bifurcation [36], is adopted the following form:

$$V(x,y) = 0.25(x^4 + y^4) - (0.55x^2 + 0.5y^2) - 0.005x^2y^2. \tag{3}$$

The 3-dimensional plot of the four-well potential is shown in Fig. 1(a). Obviously, the potential has nine equilibrium points located in the $x-y$ phase plane (see Fig. 1(b)), where there are four stable nodes (S_1, S_2, S_3, S_4), four saddle points ($U_{12}, U_{23}, U_{34}, U_{41}$), and an unstable node O. Due to the symmetry of the potential, all activation barrier energies only have two different values: $\Delta V_1 = 0.31$ and $\Delta V_2 = 0.26$. To investigate SR, the sub-threshold signal amplitudes should satisfy the condition: $\max\{A_x, A_y\} < \min\{\Delta V_1, \Delta V_2\}$. To guarantee the condition of adiabatic approximation, the modulation frequencies should be smaller than the intrawell relaxation frequencies. Thus, in the following analysis, the parameter values of the driving signals are chosen as $A_x = 0.2$, $\lambda \in [0, 1]$, $\omega_x = \pi/20$, $\eta > 0$, $\varphi_x = 1.3$, and $\Delta\varphi \in [0, 2\pi]$.

The dynamics in the deterministic systems (1) depends not only on the coupling strength, but also on the two driving signals. The maximal Lyapunov exponent calculated by Wolf algorithm [37] is displayed as a function of coupling strength c in the absence of noises in Fig. 2. When the amplitudes of two driving signals are different (see Fig. 2(a)), the systems always maintain deterministic dynamics for any value of c. However, the increasing c induces the transition from deterministic to chaotic dynamics when the two driving frequencies have a significant difference in Fig. 2(b). Moreover, since noise is ubiquitous in nature, it is worthwhile to study the systems driven by two different driving signals in the presence of noises.