Physica A 492 (2018) 1604–1624

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The global dynamics for a stochastic SIS epidemic model with isolation

Yiliang Chen, Buyu Wen, Zhidong Teng *

College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, People's Republic of China

HIGHLIGHTS

- A stochastic SIS epidemic model with isolation and multiple noises perturbation is proposed.
- The criteria on the extinction and persistence in the mean with probability one are obtained.
- The sufficient conditions for the existence of unique stationary distribution are established.

ARTICLE INFO

Article history:
Received 23 May 2017
Received in revised form 21 October 2017
Available online 21 November 2017

Keywords:
Stochastic SIQS epidemic model
Threshold value
Persistence in the mean
Extinction
Stationary distribution

ABSTRACT

In this paper, we investigate the dynamical behavior for a stochastic SIS epidemic model with isolation which is as an important strategy for the elimination of infectious diseases. It is assumed that the stochastic effects manifest themselves mainly as fluctuation in the transmission coefficient, the death rate and the proportional coefficient of the isolation of infective. It is shown that the extinction and persistence in the mean of the model are determined by a threshold value R_0^S. That is, if $R_0^S < 1$, then disease dies out with probability one, and if $R_0^S > 1$, then the disease is stochastic persistent in the means with probability one. Furthermore, the existence of a unique stationary distribution is discussed, and the sufficient conditions are established by using the Lyapunov function method. Finally, some numerical examples are carried out to confirm the analytical results.

1. Introduction

As is well-known, in the theory of epidemiology the quarantine/isolation is an important strategy for the control and elimination of infectious diseases. Such as, in order to control SARS, the Chinese government is the first to use isolation. The various types of classical epidemic models with quarantine/isolation have been investigated in many articles. See, for example [1–15] and the references cited therein.

Particularly, in [1], Herbert et al. studied the following SIS epidemic model with isolation

$$\begin{cases} S'(t) = A - \beta IS - \mu S + \gamma I + \xi Q, \\ I'(t) = \beta IS - (\mu + \gamma + \delta + \alpha)I, \\ Q'(t) = \delta I - (\mu + \xi + \alpha)Q. \end{cases} \tag{1.1}$$

where $S(t)$ denotes the number of individuals who are susceptible to an infection, $I(t)$ denotes the number of individuals who are infectious but not isolated, $Q(t)$ is the number of individuals who are isolated. A is the recruitment rate of $S(t)$, β

* Corresponding author.
E-mail addresses: zhidong_teng@sina.com, zhidong@xju.edu.cn (Z. Teng).

https://doi.org/10.1016/j.physa.2017.11.085
0378-4371/

is the transmission rate coefficient between compartment $S(t)$ and $I(t)$, μ is natural death rate of $S(t)$, $I(t)$ and $Q(t)$, α is the disease-related death rate of $I(t)$, δ is the proportional coefficient of isolated for the infection, γ and ξ are the rates where individuals recover and return to $S(t)$ from $I(t)$ and $Q(t)$, respectively. All parameters are usually assumed to be nonnegative.

In addition, we see that the quarantine/isolation strategies also are introduced and investigated in many practical epidemic model, such as the emerging infectious disease, two-strain avian influenza, childhood diseases, the Middle East respiratory syndrome, Ebola epidemics, Dengue epidemic, H1N1 flu epidemic, Hepatitis B and C, Tuberculosis, etc. See, for example [16–28] and the references cited therein.

As a matter of fact, epidemic systems are inevitably subjected to environmental white noise. Therefore, the studies for the stochastic epidemic models have more practical significance. In recent years, the stochastic epidemic models with the quarantine and isolation have been investigated in articles [29–32]. Particularly, in [29] Zhang et al. investigated the dynamics of the deterministic and stochastic SIQS epidemic model with an isolation and nonlinear incidence. The sufficient conditions on the extinction almost surely of the disease and the existence of stationary distribution of the model are established. Zhang et al. in [30] discussed the threshold of a stochastic SIQS epidemic model. The criteria on the extinction and permanence in the mean of global positive solutions with probability one are established. Besides, we also see that the stochastic persistence and the existence of stationary distribution for the various stochastic epidemic models and population models have been widely investigated. Some important recent works can been found in [33–43] and the references cited therein.

Motivated by the works [1,2,4,5,29–32], in this paper as an extension of model (1.1) we firstly assume that the disease-related death rates of isolation and no-isolation are different, respectively, denote by α_2 and α_3. Then, we further define $\mu_1 = \mu, \mu_2 = \mu+\alpha_2$ and $\mu_3 = \mu+\alpha_3$ for the convenience. It is clear that $\mu_1 \le \min\{\mu_2, \mu_3\}$. Next, we introduce randomness into model (1.1), by replacing the parameters β, $\mu_i\,(i = 1, 2, 3)$ and δ with $\beta \to \beta + \sigma_1\dot{W}_1(t)$, $\mu_2 \to \mu_2 + \sigma_2\dot{W}_2(t)$, $\mu_3 \to \mu_3 + \sigma_3\dot{W}_3(t)$, $\delta \to \delta + \sigma_4\dot{W}_4(t)$ and $\mu_1 \to \mu_1 + \sigma_5\dot{W}_5(t)$, where $W_i(t)\,(i = 1, 2, 3, 4, 5)$ are independent standard Brownian motion defined on some probability space $(\Omega, \mathcal{F}, P)$ and parameter $\sigma_i > 0$ represents the intensity of $W_i(t)$. Thus, we establish the following stochastic SIS epidemic model with multi-parameters white noises perturbations and the isolation of infection.

$$\begin{cases} dS = [A - \beta IS - \mu_1 S + \gamma I + \xi Q]dt - \sigma_1 SIdW_1(t) + \sigma_5 SdW_5(t), \\ dI = [\beta IS - (\mu_2 + \gamma + \delta)I]dt + \sigma_1 SIdW_1(t) + \sigma_2 IdW_2(t) - \sigma_4 IdW_4(t), \\ dQ = [\delta I - (\mu_3 + \xi)Q]dt + \sigma_3 QdW_3(t) + \sigma_4 IdW_4(t). \end{cases} \tag{1.2}$$

Our purpose in this paper is to study the stochastic extinction and persistence, and the stationary distribution of model (1.2). We will establish a series of sufficient conditions to assure the extinction and persistence in the mean of the model with probability one, and the existence of unique stationary distribution for model (1.2) by using the theory of stochastic processes, the Ito's formula and the Liapunov function method.

This paper is organized as follows. In Section 2, we introduce the preliminaries and some useful lemmas. In Section 3, the criteria on the extinction and persistence in the mean with probability one for model (1.2) are stated and proved. In Section 4, the criteria on the existence of a unique stationary distribution for model (1.2) are stated and proved. In Section 5, the numerical examples are carried out to illustrate the main theoretical results.

2. Preliminaries

We denote $\mathbb{R}^3_+ = \{(x_1, x_2, x_3) : x_i > 0, i = 1, 2, 3\}$. For an integrable function $f(t)$ defined on $[0, \infty)$, denote $\langle f(t)\rangle = \frac{1}{t}\int_0^t f(s)ds$.

As the preliminaries, we give the following lemmas.

Lemma 2.1. *For deterministic model (1.1), let* $R_0 = \frac{\beta A}{\mu(\delta+\gamma+\mu+\alpha)}$. *We have following conclusions.*

(1) If $R_0 < 1$, then model (1.1) has only a disease-free equilibrium $E_0(\frac{A}{\mu}, 0, 0)$, which is globally asymptotically stable.

(2) If $R_0 > 1$, then model (1.1) also has an endemic equilibrium $E^*(S^*, I^*, Q^*)$, which is globally asymptotically stable, where

$$S^* = \frac{A}{\mu R_0},\ I^* = \frac{A(1-\frac{1}{R_0})}{(\mu+\alpha)(1+\frac{\delta}{\mu+\xi+\alpha})},\ Q^* = \frac{\delta I^*}{\mu+\xi+\alpha}.$$

The proof of Lemma 2.1 can be found in [1]. We hence omit it here.

Lemma 2.2. *For any given initial value* $(S(0), I(0), Q(0)) \in \mathbb{R}^3_+$, *model (1.2) has a unique global positive solution* $(S(t), I(t), Q(t))$. *That is, solution* $(S(t), I(t), Q(t))$ *is defined for all* $t \ge 0$ *and remains in* $\mathbb{R}^3_+$ *with probability one.*

Lemma 2.2 can be proved by using the similar method given in [29].