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Hyperscaling breakdown and Ising spin glasses: The Binder cumulant

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HIGHLIGHTS

- Data on lattice Ising models are analyzed from criticality to infinite temperature.
- For the model in dimension 3, hyperscaling rules hold over all this range.
- In dimension 5, scaling with known hyperscaling breakdown holds at all temperatures.
- In Spin Glasses, Binder cumulant hyperscaling breaks down due to strong disorder.

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ABSTRACT

Among the Renormalization Group Theory scaling rules relating critical exponents, there are hyperscaling rules involving the dimension of the system. It is well known that in Ising models hyperscaling breaks down above the upper critical dimension. It was shown by Schwartz (1991) that the standard Josephson hyperscaling rule can also break down in Ising systems with quenched random interactions. A related Renormalization Group Theory hyperscaling rule links the critical exponents for the normalized Binder cumulant and the correlation length in the thermodynamic limit. An appropriate scaling approach for analyzing measurements from criticality to infinite temperature is first outlined. Numerical data on the scaling of the normalized correlation length and the normalized Binder cumulant are shown for the canonical Ising ferromagnet model in dimension three where hyperscaling holds, for the Ising ferromagnet in dimension five (so above the upper critical dimension) where hyperscaling breaks down, and then for Ising spin glass models in dimension three where the quenched interactions are random. For the Ising spin glasses there is a breakdown of the normalized Binder cumulant hyperscaling relation in the thermodynamic limit regime, with a return to size independent Binder cumulant values in the finite-size scaling regime around the critical region.

1. Introduction

The consequences of the Renormalization Group Theory (RGT) approach have been studied in exquisite detail in numerous regular physical models, typified by the canonical near-neighbor interaction ferromagnetic Ising models. It has been tacitly assumed that Edwards–Anderson Ising Spin Glasses (ISGs), where the quenched interactions are random, follow the same basic scaling and Universality rules as the Ising models.

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The Binder cumulant [1] is an important observable which has been almost exclusively exploited numerically for its scaling properties as a dimensionless observable very close to criticality in the finite-size scaling (FSS) regime $L \ll \xi(\beta)$, where L is the sample size and $\xi(\beta)$ is the second-moment correlation length at inverse temperature β. Here we will consider its scaling properties over the whole temperature region, in particular in the Thermodynamic limit (ThL) regime $L \gg \xi(\beta)$ where the properties of a finite-size sample normalized appropriately are independent of L and so are the same as those of the infinite-size model.

We will explain in detail the overall scaling analysis procedure, based on Ref. [2–4], which we use in both the cases of standard Ising models and of ISGs.

2. Scaling

In numerical simulation analyses the conventional RGT based approach consists in using as the thermal scaling variable the reduced temperature $t = (T - T_c)/T_c$, together with the principal observables $\chi(t, L)$ the susceptibility, $\xi(t, L)$ the second moment correlation length, and $g(t, L)$ the Binder cumulant. (For finite-size simulation data the standard finite-L definition for the second moment correlation length $\xi(\beta, L)$ through the Fourier transformation of the correlation function is used, see for instance Ref. [5] Eq. 14). The conventional approach is tailored to the critical region; however at high temperatures t diverges and $\xi(t, L)$ tends to zero, so it is not possible to analyze the entire paramagnetic regime without introducing diverging correction terms. For the Ising systems this problem can be eliminated by using the inverse temperature $\beta = 1/T$, a practice which pre-dates RGT.

The thermal scaling variable t is also widely used in analyses of simulation data in ISGs. As the relevant interaction strength in ISGs is $[\langle J_{ij}^2\rangle]$, the symmetric interaction distribution ISG thermal scaling variable should logically depend on the square of the temperature; this basic point was made some thirty years ago [6] but has generally been ignored.

As a basis for a rational scaling approach which englobes the entire paramagnetic region so including both the finite-size scaling regime (FSS, $L \ll \xi(\beta, \infty)$) and the thermodynamic-limit regime (ThL, $L \gg \xi(\beta, \infty)$), we start from the Wegner ThL scaling expression for the Ising susceptibility [2]

$$\chi(\tau) = C_\chi \tau^{-\gamma} \left(1 + a_\chi \tau^\theta + b_\chi \tau + \cdots\right) \tag{1}$$

where $\tau = 1 - \beta/\beta_c$ with β the inverse temperature. (The Wegner expression is often mis-quoted with t replacing τ). The terms inside (..) are scaling corrections, with θ the leading correction exponent which is universal for all observables. As τ and $\chi(\tau)$ both tend to 1 at infinite temperature, the whole paramagnetic region can be covered without divergencies, to good precision when a small number of well-behaved correction terms are included. (To obtain infinite precision an infinite number of correction terms would be needed, just as in standard FSS analyses perfect precision in principle requires a series of corrections to infinite L). In ISG models where the interaction distributions are symmetric about zero, an appropriate thermal scaling variable to be used with the same Wegner expression is $\tau = 1 - (\beta/\beta_c)^2$, Refs. [4,6–8]. In the ThL regime $L \gg \xi(\beta)$ the properties of a finite-size sample, if normalized correctly, are independent of L and so are the same as those of the infinite-size model. A standard rule of thumb for the approximate onset of the ThL regime is $L > 7\xi(\beta, L)$ and the ThL regime can be easily identified in simulation data. An important virtue of this approach is that the ThL numerical data can be readily dovetailed into High Temperature Series Expansion (HTSE) values calculated from sums of exact series terms (limited in practice to a finite number of terms). No such link can be readily made when the conventional FSS thermal scaling variable t is used.

To apply the Wegner formalism to observables Q other than χ, we introduce the rule that these observables should be normalized in such a way that the infinite-temperature limit $Q(\tau = 1) \equiv 1$, without the critical limit being modified. For the susceptibility with the standard definition no normalization is required as this condition is automatically fulfilled, with a temperature-dependent effective exponent $\gamma(\tau) = \partial \ln \chi(\tau, L)/\partial \ln \tau$ in Ising models and in ISGs with the appropriate τ. Then

$$\gamma(\tau) = \gamma - \frac{a_\chi \theta \tau^\theta + b_\chi \tau}{1 + a_\chi \tau^\theta + b_\chi \tau} \tag{2}$$

to second order in the corrections [9].

In Ref. [4] the normalized second-moment correlation length was introduced : $\xi(\tau, L)/\beta^{1/2}$ in Ising models and $\xi(\tau, L)/\beta$ in ISG models. From exact and general HTSE infinite-temperature limits, this normalized correlation length tends to exactly 1 at infinite temperature [8,10]. The temperature-dependent effective exponent is $\nu(\tau) = \partial \ln[\xi(\tau, L)/\beta^{1/2}]/\partial \ln \tau$ in Ising models and $\nu(\tau) = \partial \ln[\xi(\tau, L)/\beta]/\partial \ln \tau$ in ISG models. A Wegner-like relation is

$$\xi(\tau)/\beta^{1/2} = C_\xi \tau^{-\nu} \left(1 + a_\xi \tau^\theta + b_\xi \tau + \cdots\right) \tag{3}$$

so

$$\nu(\tau) = \nu - \frac{a_\xi \theta \tau^\theta + b_\xi \tau}{1 + a_\xi \tau^\theta + b_\xi \tau}. \tag{4}$$

The critical limiting ThL exponent ν is unaltered by this normalization (models with zero critical temperatures are a special case). The normalized correlation length can be accurately expressed over the entire paramagnetic region with a limited