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Lie symmetry analysis, conservation laws and exact solutions of the time-fractional generalized Hirota–Satsuma coupled KdV system

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HIGHLIGHTS

- Symmetry operators of the time-fractional generalized Hirota–Satsuma coupled KdV system are given.
- Reduced equations of the time-fractional generalized Hirota–Satsuma coupled KdV system are computed.
- Conservation laws of the considered system are obtained.
- Some exact solutions are derived via invariant subspace method.

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ABSTRACT

In the present paper, Lie point symmetries of the time-fractional generalized Hirota–Satsuma coupled KdV (HS-cKdV) system based on the Riemann–Liouville derivative are obtained. Using the derived Lie point symmetries, we obtain similarity reductions and conservation laws of the considered system. Finally, some analytic solutions are furnished by means of the invariant subspace method in the Caputo sense.

1. Introduction

The idea of fractional calculus and fractional order differential equations and inclusions has been a subject of interest not only among mathematicians but also among physicists and engineers. Indeed, we can find numerous applications in rheology, porous media, viscoelasticity, electrochemistry, electromagnetism, signal processing, optics, geology, medicine, economics, probability and statistics, astrophysics, chemical engineering, fluid mechanics, nonlinear control, chaotic dynamics, polymer science, neural networks, etc.

Numerous decomposition numerical methods such as Adomian decomposition method [1,2], new iterative method [3], first integral method [4], homotopy perturbation method [5], have been employed to solve FDEs. The solutions obtained by all these methods however are local in nature and it is important to explore other techniques in order to find exact analytical solutions of FDEs. Exact solutions play a vital role in the proper understanding of qualitative features of the concerned phenomena and processes in various areas of science and engineering.

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Lie symmetry analysis is one of the most general and effective methods for obtaining exact solutions of nonlinear partial differential equations (PDEs). This method was introduced by Sophus Lie more than one hundred years ago. However, application of the Lie symmetry analysis to FPDEs is quite new. Rigorous study of symmetries, admitted by such fractional equations, was started by Gazizov and his collaborators in [6] that they proposed prolongation formulae for two basic fractional derivatives: Riemann–Liouville and Caputo.

One of the analytical methods for finding exact solutions of PDEs is the invariant subspace method. This method was initially proposed by Galaktionov and Svirshchevskii [7,8]. Recently, the effectiveness of the invariant subspace method has been extended for time fractional PDEs by many authors [9,10]. In this method, FPDEs are reduced to systems of FODEs which can be solved by known analytical methods. It is appropriate to mention that the invariant subspace method has been extended to time fractional coupled nonlinear PDEs and constructs their exact solutions [10].

One the other hand, as stated in [11] conservation laws plays an very important role in the analysis of essential properties of the solutions, particularly, investigation of existence, uniqueness and stability of solutions [12,13]. There are many methods of constructing conservation laws for DEs; One of the most well-known systematic methods is the Noether's theorem [14]. In [15–19], the authors have provided fractional generalizations of Noether's theorem in order to find conservation laws of FDEs. However, many FDEs do not admit fractional Lagrangians. On the basis of new conservation law theorem firstly proposed by Ibragimov [20], Lukashchuk proposed the fractional generalizations of the Noether operators without fractional Lagrangians and derived conservation laws for arbitrary time-fractional PDEs [21].

In this article, we focus on the following time-fractional generalized Hirota–Satsuma coupled KdV system of the form:

$$\begin{aligned} \partial_t^\alpha u &= \frac{1}{2}u_{xxx} - 2uu_x + 3wv_x + 3vw_x, \\ \partial_t^\alpha v &= -v_{xxx} + 3uv_x, \\ \partial_t^\alpha w &= -w_{xxx} + 3uw_x, \end{aligned} \tag{1}$$

where ∂_t^α is the fractional derivative of order α with $0 < \alpha < 1$ and $u, v, w(x,t)$ are the amplitude of the wave modes as functions of space x and time t. The Hirota–Satsuma system was introduced to describe the interaction of long waves with different dispersion relations [22]. Namely, it is connected with most types of long waves with weak dispersion, internal, acoustic, and planetary waves in geophysical hydrodynamics [23]. The case when $\alpha = 1$ in system (1) was introduced by Wu et al. [24]. The HS-cKdV system of fractional order has been discussed analytically and numerically by many researchers, for example, Ganji et al. have shown in [25,26] how to obtain soliton solutions of (1). A numerical method is proposed for the time-fractional generalized Hirota–Satsuma coupled KdV system by Abazari et al. [27], they used the reduced differential transform method. The first integral method is employed for constructing the exact solutions of the time-fractional generalized HS-cKdV system by Lu [28]. Martínez, Reyes and Sosa [29] applied sub-equation method to the space–time fractional generalized HS-cKdV system. Bekir et al. obtained the exact solutions via sub-equation method for the time-fractional generalized HS-cKdV system [30]. Li examined the approximate analytical solutions of Caputo time fractional-order HS-cKdV system via generalized two-dimensional differential transform method [31], Merdan used the variational iteration method to solve (1) with modified Riemann–Liouville derivative [32].

In the present work, we study symmetry reductions and conservation laws of the time-fractional HS-cKdV system in the Riemann–Liouville sense with the help of Lie symmetry analysis and Ibragimov's non local conservation method [21,33], respectively. Furthermore, the solutions of the HS-cKdV system with time-fractional derivatives has been found by employing invariant subspace method in the Caputo sense. Some of the solutions are plotted in the paper.

2. Fractional calculus

Before embarking into the details of the Lie symmetry method for FDEs, we give some basic definitions, notations and properties of the fractional calculus theory that are used in this work. See [34–36] for more details.

The Riemann–Liouville fractional integral operator of order $\alpha > 0$ of the function $f(t) \in \mathrm{L}^1([a,b], \mathbb{R}_+)$ is defined by:

$$\mathrm{J}_a^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\zeta)^{\alpha-1} f(\zeta) d\zeta, \qquad t > a, \tag{2}$$

where $\Gamma(z)$ is the Euler gamma function, that is,

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \quad \mathrm{Re}(z) > 0.$$

By definition, we have $\mathrm{J}_a^0 f(t) = f(t)$ and it satisfies the stability property $\mathrm{J}_a^\alpha \mathrm{J}_t^\beta f(t) = \mathrm{J}_a^{\alpha+\beta} f(t)$.

Definition 2.1. The Riemann–Liouville fractional derivative is defined by

$${}_a^R D_t^\alpha f(t) = D_t^n \mathrm{J}_a^{n-\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\partial^n}{\partial t^n} \int_a^t (t-\zeta)^{n-\alpha-1} f(\zeta) d\zeta, \qquad \alpha \neq n, \tag{3}$$

wherein $0 \le n-1 < \alpha \le n$, $n \in \mathbb{Z}^+$.