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# Average inactivity time model, associated orderings and reliability properties

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## HIGHLIGHTS

- We introduce and study a new model called 'average inactivity time model'.
- The model handles the heterogeneity of the time of the failure of a system.
- Bounds for mean average inactivity time are derived under the setting of the model.
- Dependencies between the average and the mixing variables in the model are identified.
- By the conception of the model a new stochastic order is proposed.
- The model has been explained by some reliability problems.

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## ABSTRACT

In this paper, we introduce and study a new model called 'average inactivity time model'. This new model is specifically applicable to handle the heterogeneity of the time of the failure of a system in which some inactive items exist. We provide some bounds for the mean average inactivity time of a lifespan unit. In addition, we discuss some dependence structures between the average variable and the mixing variable in the model when original random variable possesses some aging behaviors. Based on the conception of the new model, we introduce and study a new stochastic order. Finally, to illustrate the concept of the model, some interesting reliability problems are reserved.


## 1. Introduction

The inactivity time (IT) represents the interval time elapsed after an event occurs until the time of its observation, and it is useful to predict the exact times of occurrence of events. Even if the IT has been mainly used in reliability, it has also been useful to describe the behavior of lifetime random variables in survival retrospective studies and some of its applications have been derived in risk theory, and econometrics. The mean inactivity time (MIT) function is used to characterize the underlying distribution which has found several applications in many disciplines such as reliability theory, survival analysis and actuarial studies (cf. Chandra and Roy [1] and Finkelstein [2]). Based on the MIT function, various types of stochastic orders and their reliability properties have been developed rapidly over the years, resulting in a large body of literature

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(cf. Nanda et al. [3], Kayid and Ahmad [4], Ahmad and Kayid [5] and Kayid and Izadkhah [6]). Stochastic comparisons of coherent systems according to their IT have been considerably studied in the literature (cf. Li and Lu [7], Gupta et al. [8], Hazra and Nanda [9] and Gupta [10], among many others).

Let $X$ be a non-negative continuous random variable having support $S_X = (l_X, u_X)$ where $l_X = \inf\{x : F(x) > 0\}$ and $u_X = \sup\{x : F(x) < 1\}$, and let $F$ be the cumulative distribution function (cdf) of $X$. Then, the conditional random variable

$$X_{(t)} = [t - X \mid X \leq t], \text{ for any } t > l_X,$$

is termed as the inactivity time of an item with original lifetime $X$ at time point $t$ provided that the item has failed before the time $t$. In the sequel, we use increasing and decreasing in place of non-decreasing and non-increasing, respectively. The random variable $X_{(t)}$ for all $t$ for which $F(t) > 0$ has cdf

$$F_{(t)}(a) = \begin{cases} 0, & a < 0 \\ 1 - \dfrac{F(t-a)}{F(t)}, & a \geq 0. \end{cases} \tag{1.1}$$

The MIT of $X$ is defined as $m_X(t) = E(X_{(t)})$ which is given by (cf. Kayid and Ahmad [4])

$$m_X(t) = \begin{cases} \displaystyle\int_0^t \frac{F(x)}{F(t)} dx, & t > l_X \\ 0, & t \leq l_X. \end{cases} \tag{1.2}$$

Another interesting reliability measure is reversed hazard rate (RHR) function which is closely related to the concept of IT defined as the ratio of the density function, if it exists, to its corresponding distribution function. Formally, the RHR function of the random variable $X$ which is assumed to have an absolutely continuous distribution function $F$ and probability density function (pdf) $f$ is given by

$$\begin{aligned} r_X(t) &= \lim_{a \to 0^+} a^{-1} F_{(t)}(a) \\ &= \frac{f(t)}{F(t)}, \quad t > l_X. \end{aligned} \tag{1.3}$$

Consider two absolutely continuous random variables $X$ and $Y$ with supports $(l_X, u_X)$ and $(l_Y, u_Y)$, cdf's $F$ and $G$, respectively. Next, the definitions of the usual stochastic order ($\leq_{ST}$), the reversed hazard rate order ($\leq_{RHR}$) and the mean inactivity time order ($\leq_{MIT}$) are recalled from Kayid and Ahmad [4] and Shaked and Shanthikumar [11].

**Definition 1.1.** The random variable $X$ is said to be smaller than $Y$ in the:

(i) usual stochastic (ST) order (denoted as $X \leq_{ST} Y$) if, and only if, $F(t) \geq G(t)$ for all $t$.
(ii) reversed hazard rate (RHR) order (denoted as $X \leq_{RHR} Y$) if $X_{(t)} \geq_{st} Y_{(t)}$, for all $t \geq 0$,
(iii) mean inactivity time (MIT) order (denoted as $X \leq_{MIT} Y$) if and only if, $E[X_{(t)}] \geq E[Y_{(t)}]$, for all $t \geq 0$, i.e., if and only if

$$\frac{\int_0^t F(u)du}{\int_0^t G(u)du} \text{ is non-increasing in } t \in R^+.$$

Related to the above orderings, two classes of life distributions of decreasing reversed hazard rate (DRHR) and increasing mean inactivity time (IMIT) have been introduced and studied in the literature.

**Definition 1.2.** The non-negative random variable $X$ is said to be:

(i) decreasing reversed hazard rate (denoted as $X \in$ DRHR) if, and only, if its RHR function $r_X$ is decreasing on the positive real line.
(ii) increasing mean inactivity time (denoted as $X \in$ IMIT) if, and only, if its MIT function $m_X$ is increasing on the positive real line.

The purpose of this investigation is to establish a new model useful to represent the average inactivity time (AIT) of systems lifetimes via a distribution function and its reliability features. The rest of the paper is organized as follows. The precise definition of the new model and its representation are described in Section 2. In Section 3, we provide some characterization properties and discuss some dependence structures between the average variable and the mixing variable in the model when original random variable possesses some aging behaviors. In Section 4, stochastic comparisons between two models of this kind will be made to highlight the role of the factors on the stochastic variation of the model. In Section 5, a possible extension of the model to the multivariate case is proposed. Finally in Section 6, we conclude the paper with some remarks of current research.

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