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Different non-extensive models for heavy-ion collisions

Ke-Ming Shen [a,*], Tamas S. Biro [b], En-Ke Wang [a]

[a] *Key Laboratory of Quark & Lepton Physics and Institute of Particle Physics, Central China Normal University, Wuhan, China*
[b] *Institute for Particle and Nuclear Physics, Wigner Research Center for Physics, Hungarian Academy of Sciences, Budapest, Hungary*

HIGHLIGHTS

- The whole script is re-written in a better manner.
- The generating function $G(t)$ is corrected and expanded more clearly in the revised version, including the exact derivations.
- The average occupancy parameter, f, is nicely described in the text as well as other quantities applied in the manuscript.
- Figures are re-plotted in a clearer and more easily distinguishable way.

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ABSTRACT

The transverse momentum (p_T) spectra from heavy-ion collisions at intermediate momenta are described by non-extensive statistical models. Assuming a fixed relative variance of the temperature fluctuating event by event or alternatively a fixed mean multiplicity in a negative binomial distribution (NBD), two different linear relations emerge between the temperature, T, and the Tsallis parameter $q-1$. Our results qualitatively agree with that of G. Wilk. Furthermore we revisit the "Soft+Hard" model, proposed recently by G.G. Barnaföldi et.al., by a T-independent average p_T^2 assumption. Relevance of these two parameters is demonstrated. Finally we compare results with those predicted by another deformed distribution, using Kaniadakis' κ parametrization.

1. Introduction

Non-extensive statistics, as the simplest and most natural generalization of the canonical Boltzmann–Gibbs statistics, can be applied widely. Heavy-ion collisions also offer an excellent field for such applications. The transverse momentum spectra exhibit power-law-tailed behavior, that can be described with non-extensive distribution functions better than with Boltzmann distributions, $\exp(-\beta E) \to \exp_q(-\beta E)$. Here the notion of q-exponential, $e_q(x)$, stands for

$$e_q(x) := [1+(1-q)x]^{\frac{1}{1-q}}. \tag{1.1}$$

This form was first suggested by V. Pareto in 1896 [1] for describing the distribution of wealth, and it has been recently promoted by C. Tsallis [2] in connection with non-extensive entropy. The use of a cut power-law in high energy physics was first introduced by R. Hagedorn [3], describing the data of the invariant cross section of hadrons as a function of p_T over a wide range. Moreover, C. Michael also proposed a power-law distribution to investigate the p_T-dependence in [4].

In [5] it is pointed out that any distribution can be mathematically derived from entropy maximization if the constraints are chosen accordingly. As trivial it sounds, it nevertheless implies that the variational formalism of q-deformed distributions

* Corresponding author.
E-mail address: shenkm@mails.ccnu.edu.cn (K.-M. Shen).

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is essentially insubstantial when fitting the p_T spectrum. Rather a physical foundation should be given to the dynamical process resulting in the optimal distribution: Investigations on the trends in terms of T and q are more revealing than a pure fit of their values.

In heavy-ion collisions, the final physical process, called "fragmentation", which forms the hadrons cannot be derived from basic QCD formulas because it is a soft QCD process involving virtuality and high powers of the coupling. The coupling is not asymptotic in this case. Therefore, in order to gain an improved insight into this generalized non-extensive statistics, as well as to value its application to p_T spectra on the top of pQCD motivated power-laws, ever used for high-energy jets, the meaning of its key parameter, q, must inevitably be studied.

More and more experimental data have been proved to display a power-law tailed distribution, among them the Tsallis–Pareto q-exponential distribution fits best at all p_T range. The quantity $(q-1)$ describes the departure from the Boltzmann–Gibbs thermal equilibrium statistics. The source of such deviations can be fluctuations of temperature event-by-event or finite size induced statistical correlations in the real physical systems. Considering the thermodynamical properties of hot and dense nuclear matter formed in heavy-ion collisions, it is therefore worth to study the fluctuations of the relevant physical observables, with a large number of particles produced. For example the multiplicity fluctuations in heavy-ion collisions have been investigated by several authors, including G. Wilk et al. [6]. Connecting to such studies, we have obtained Tsallis' distribution for particular particle number fluctuation patterns due to a finite heat bath, for details see [7].

Moreover, further non-extensive distribution functions were being proposed and studied to describe the data on p_T spectra in heavy-ion collisions. K. Urmossy et al. proposed a non-extensive model in which the produced hadrons in heavy-ion collisions stem from a quark–gluon plasma (QGP), referred to as 'soft' yields, on the top of those stemming from jets, called 'hard' yields [8]. Both of these contributions follow respective cut power-laws, $\exp_q(-\beta E)$, however with different values of q. Based on this view in this paper we investigate the scatter of the parameter q with respect to the fitted temperatures T. This investigation should differentiate between hard and soft origin of power laws and should look different for heavy-ion as for pp data. We shall see different behaviors of systems on the $T-q$ plane in view of the fact that randomness plays a role in fragmentation in heavy-ion collisions. Since statistical models assume it does, then its result must be related to information entropy. Considering the information entropy it is not a new habit to look beyond the Boltzmann–Gibbs–Planck–Shannon formula.

Considering quantum-statistics even further constraints arise from particle–hole CPT symmetry. They lead to a special requirement on the cut power-law function, alternatively called deformed exponential [9]:

$$e_q(x) \cdot e_q(-x) = 1. \tag{1.2}$$

For the Pareto–Hagedorn–Tsallis distribution, $\exp_q(x)$, it is easy to see that it does not satisfy this relation, therefore it is inadequate to apply naively extended quantum statistical distributions in the form

$$n_{B,F} = \frac{1}{e_q(x) \mp 1} \tag{1.3}$$

with $x = (\omega - \mu)/T$, where ω, μ and T are the energy, chemical potential and temperature of the system, respectively, following the notation used in [10]. In the present paper we shall also apply the κ-exponential distribution, proposed by G. Kaniadakis for the description of relativistic plasmas [11]. We compare our corresponding analysis with the ones using Tsallis' distribution, discussed above. The κ-exponential function has the advantage that it readily satisfies the particle–hole symmetry demand:

$$e_\kappa(x) \cdot e_\kappa(-x) = 1, \tag{1.4}$$

based on the definition

$$e_\kappa(x) := [\sqrt{1+(\kappa x)^2} + \kappa x]^{\frac{1}{\kappa}}. \tag{1.5}$$

2. p_T spectra with finite heat capacity

Particle transverse momentum (p_T) spectra provide a tool for measuring thermal properties of the QGP formed in ultra-relativistic heavy-ion collisions. Nowadays more and more experimental data show that the p_T spectrum exhibits power-like rather than the earlier expected exponential behavior, and multiparticle distributions are also broader than expected [12]. The number of particles in such processes is not that large ($N \sim 10^3 - 10^5 \ll 10^{23}$, much less than Avogadro's number). All of these warrant a suitable modification of the thermal model by accounting for possible intrinsic fluctuations in observables measured in high-energy collisions.

The non-extensive form of statistical mechanics, proposed by Tsallis [2], has found applications in many fields including high energy physics [13,14]. In several situations the p_T spectra can be best described by a Tsallis distribution, characterized by a non-extensive parameter q and a scale parameter T [15]. G. Wilk et al. have demonstrated that q turns to be a function of fluctuations of temperature T, together with fluctuations of other variables, from an analysis of different experimental