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Optimal harvesting of a stochastic delay tri-trophic food-chain model with Lévy jumps

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HIGHLIGHTS

- A stochastic delay tri-trophic food-chain model with Lévy jumps.
- The critical values between extinction and persistent in the mean of each species are established.
- The sufficient and necessary criteria for the existence of optimal harvesting policy are established.
- The optimal harvesting effort and the maximum of sustainable yield are obtained.
- White noises and Lévy noises significantly affect the optimal harvesting policy.

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ABSTRACT

In this paper, the optimal harvesting of a stochastic delay tri-trophic food-chain model with Lévy jumps is considered. We introduce two kinds of environmental perturbations in this model. One is called white noise which is continuous and is described by a stochastic integral with respect to the standard Brownian motion. And the other one is jumping noise which is modeled by a Lévy process. Under some mild assumptions, the critical values between extinction and persistent in the mean of each species are established. The sufficient and necessary criteria for the existence of optimal harvesting policy are established and the optimal harvesting effort and the maximum of sustainable yield are also obtained. We utilize the ergodic method to discuss the optimal harvesting problem. The results show that white noises and Lévy noises significantly affect the optimal harvesting policy while time delays is harmless for the optimal harvesting strategy in some cases. At last, some numerical examples are introduced to show the validity of our results.

1. Introduction

Establishing the environmentally, ecologically and economically reasonable harvesting strategies in natural resources management have increased significantly in recent years(e.g. Song et al. [1]). As far as we know, with the rapid development of economy and society, unreasonable harvesting policies and over-harvesting have resulted in a great many detrimental impacts (e.g. Lande et al. [2]). Hence, it is significant and meaningful to research the optimal harvesting strategies for the ecology, environment, society, and the economy.

In recent years, determining the optimal harvesting strategies has received much attention. And several authors have concentrated their research work in various deterministic systems, see e.g., [3–7]. However, in virtually all ecosystems, the

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population dynamics are inevitably affected by some environmental perturbations, which mainly being considered as the following two types:

(i) White noise, which describes the continuous noise, such as drought, cold wave, fire and so on. Qiu and Deng [8] discussed some dynamical properties of a three-species stochastic food-chain system and Liu and Bai [9] studied a stochastic tri-trophic food-chain model with harvesting. Liu and Bai [10] also considered the optimal harvesting of a stochastic delay competitive model.

(ii) Jumping noise, such as earthquakes, floods, epidemics and so on. [11,12] have introduced that Lévy process can describe the sudden environment shocks. Zhao and Yuan [13] discussed the stability in distribution of a stochastic hybrid competitive Lotka–Volterra model with Lévy jumps. Liu and Bai [14] investigated the dynamics of a stochastic one-prey two-predator model with Lévy jumps. And the optimal harvesting problem for a stochastic logistic jump-diffusion process was studied in Zou and Wang [15].

On the other hand, we should consider a time delay model. The feature of a time delay model is that the system's future evolution depends not only on its present state, but also on a period of its history. While we all know that for a deterministic delay population model, if the model is a "no-pure-delay-type", i.e. "system contains delay independent terms which dominate other intraspecific and interspecific interaction effects with delays", then the delay is harmless for the global stability of positive steady state of the model (see Zhen and Ma [16]). However, there are times when delay effect cannot be neglected, or it will be more beneficial for our model to be accounted for in the nature. Ruan [17] have showed that time delay is important in ecosystem model and Gopalsamy [18] have pointed out that "the current growth of a population should also be influenced by the past history of the species". So it is necessary to take time delay into consideration. Qiu and Deng [19] discussed the optimal harvesting problem of a stochastic delay logistic model with Lévy jumps, which is a single-species model. But in the nature, many species interact with each other and compete for food, habitat or territory. Therefore, it is more natural and practical to consider the harvesting problems in multi-species models both with environmental perturbations and time delays. One of the most important population models is tri-trophic food-chain ecosystems. Liu and Bai [9] considered a tri-trophic stochastic food-chain model with harvesting and showed that the white noises can reduce the optimal harvesting. Then an important and significant problem is that: if consider the optimal harvesting of a tri-trophic stochastic food-chain model with white noises, Lévy noises and time delays, what are the effects of these environmental perturbations and time delays on optimal harvesting? Therefore, in this paper, we consider the optimal harvesting strategy for a stochastic delay three-species food-chain model with Lévy jumps.

2. Problem formulation

As previously introduced, a reasonable stochastic delay three-species food-chain model both with Lévy jumps and harvesting is that:

$$\begin{cases} dx_1(t) = x_1(t^-)\big[r_1 - h_1 - c_{11}x_1(t^-) - c_{12}x_2(t-\tau_1)\big]dt + \alpha_1 x_1(t^-)dB_1(t) \\ \qquad\qquad + x_1(t^-)\displaystyle\int_{\mathbb{Z}} \gamma_1(v)\widetilde{N}(dt,dv), \\ dx_2(t) = x_2(t^-)\big[-r_2 - h_2 + c_{21}x_1(t-\tau_2) - c_{22}x_2(t^-) - c_{23}x_3(t-\tau_3)\big]dt \\ \qquad\qquad + \alpha_2 x_2(t^-)dB_2(t) + x_2(t^-)\displaystyle\int_{\mathbb{Z}} \gamma_2(v)\widetilde{N}(dt,dv), \\ dx_3(t) = x_3(t^-)\big[-r_3 - h_3 + c_{32}x_2(t-\tau_4) - c_{33}x_3(t^-)\big]dt \\ \qquad\qquad + \alpha_3 x_3(t^-)dB_2(t) + x_3(t^-)\displaystyle\int_{\mathbb{Z}} \gamma_3(v)\widetilde{N}(dt,dv), \end{cases} \tag{1}$$

with initial data

$$x_i(\theta) = \phi_i(\theta), \quad \theta \in [-\tau, 0], \quad \tau = \max\{\tau_1, \tau_2, \tau_3, \tau_4\}, \quad i = 1, 2, 3,$$

where x_i stands for the population size of each species, and m_0 denotes the units of mass of the organism, $i = 1, 2, 3$; $r_1 > 0$ is the growth rate of x_1, $-r_i > 0$ is the growth rate of x_i, $i = 2, 3$, and the unit of r_i is t^{-1}, $i = 1, 2, 3$; $h_i > 0$ is the harvesting effort (in units: t^{-1}) of x_i, $i = 1, 2, 3$; $c_{ii} > 0$ is the intraspecific competition coefficients of x_i, i=1,2,3; $c_{12} > 0$, $c_{23} > 0$ denote the capture rates; $c_{21} > 0$, $c_{32} > 0$ denote the efficiency of food conversion; the unit of c_{ij} is $t^{-1}m_0^{-1}$, $i, j = 1, 2, 3$; $\tau_i \geqslant 0$, $i = 1, 2, 3, 4$ are time delays; $\phi_i(\theta) > 0$, $i = 1, 2, 3$ are continuous functions defined on $[-\tau, 0]$. α_i^2, $i = 1, 2, 3$ denote the intensity of the white noise. $B_i(t)_{t\geqslant 0}$ is standard independent Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{F}_{t\geq 0}, \mathscr{P})$. $x_i(t^-)$ is the left limit of $x_i(t)$, $i = 1, 2, 3$; $\widetilde{N}(dt, dv) = N(dt, dv) - \mu(dv)dt$, N is a Poisson counting measure, μ is the characteristic measure of N on a measurable subset $\mathbb{Z}$ of $(0, +\infty)$ with $\mu(\mathbb{Z}) < +\infty$; $\gamma_i(v)$ measures the effect of Lévy noises on species. If $\gamma_i(v) > 0$, the jumps represent the increasing of the species (e.g., planting); if $\gamma_i(v) < 0$, the jumps represent the decreasing of the species (e.g., epidemics); Therefore, from the biological significance, it is reasonable to assume that $1 + \gamma_i(v) > 0$, $v \in \mathbb{Z}$, $i = 1, 2, 3$.

In this paper, our main purposes is to get the optimal harvesting effort (OHE) $H^* = (h_1^*, h_2^*, h_3^*)$ such that the expectation of sustainable yields (ESY) $Y(H) = \lim_{t\to+\infty}\sum_{i=1}^{3}\mathbb{E}(h_i x_i(t))$ (in units: $t^{-1}m_0$) is maximum (under the condition of all species are persistent) and analyze how do white noises, Lévy noises and time delays affect the optimal harvesting strategy.