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Stochastic persistence and stationary distribution in an SIS epidemic model with media coverage

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HIGHLIGHTS

- A stochastic epidemic model with media coverage is developed.
- The global dynamics of the deterministic model is shown.
- The stochastic dynamics of the SDE model is given.
- The existence of a unique stationary distribution of the SDE model is displayed.

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ABSTRACT

This paper aims to study an SIS epidemic model with media coverage from a general deterministic model to a stochastic differential equation with environment fluctuation. Mathematically, we use the Markov semigroup theory to prove that the basic reproduction number $\mathcal{R}_0^s$ can be used to control the dynamics of stochastic system. Epidemiologically, we show that environment fluctuation can inhibit the occurrence of the disease, namely, in the case of disease persistence for the deterministic model, the disease still dies out with probability one for the stochastic model. So to a great extent the stochastic perturbation under media coverage affects the outbreak of the disease.

1. Introduction

When an infectious disease appears and spreads in one place, all possible effective ways to prevent the disease will be taken by the departments for disease control and prevention. A rapid and timely measure is through the media coverage to tell people how to prevent the spread of the disease [1–3]. Media education plays a vital role in controlling the spread of the disease. As we know, one big characteristic of the infectious disease is the infectiousness, namely the pathogens of infectious disease, can spread from an infected person to a susceptible person through a certain way. The spread ways of the infectious disease are not the same, and its communication process is influenced by natural factors and social factors [4,5]. When an infectious disease appears, if we can timely find its route of transmission, and encourage people to learn relevant publicity and education of the disease, thus can effectively control the outbreak of the disease. Media coverage may reduce the contact rate of people, which has been found during the spreading of severe acute respiratory syndrome (SARS) in 2003 [6–8].

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Assume that the total population N is divided into two groups, susceptible (uninfected) S and infected I, i.e., $N = S + I$. Then the dynamics of the disease transmission can be governed by the classical SIS epidemic model with mass action incidence rate as follows:

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - \mu S - \beta SI + \gamma I, \\ \dfrac{dI}{dt} = \beta SI - (\mu + \gamma)I, \end{cases} \tag{1}$$

where parameters Λ, μ, γ and β are all positive constants. Λ is the recruitment rate of the population, μ the natural death rate of the population, γ the recovery rate of infective individuals, β the contact transmission coefficient. The infectious force βSI in (1) plays a key role in determining the transmission of the disease.

In fact, β depends on the number as well as the pattern of the contact between the susceptible and infected individuals. When the media coverage is intervened, the contact rate may reduce if people know about the transmission way from media and then protect themselves. Generally, from the practical significance we know that the contact rate of susceptible and infectious individuals is a decreasing function. Motivated by Cui et al. [1], we adopt the contact transmission coefficient β as

$$\beta = \beta_1 - \beta_2 f(I),$$

where β_1 is the usual contact rate without taking the infected individuals into account, and β_2 is the maximum reduced contact rate due to the presence of the infected individuals. However, we know that anyone cannot avoid contact with others, so we assume that $\beta_1 > \beta_2$. The function $f(I)$ satisfies

(H1) $f(0) = 0, f'(I) \geq 0$ and $\lim_{I\to\infty} f(I) = 1$.

Then model (1) can be rewritten as follows:

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - \mu S - (\beta_1 - \beta_2 f(I))SI + \gamma I, \\ \dfrac{dI}{dt} = (\beta_1 - \beta_2 f(I))SI - (\mu + \gamma)I, \end{cases} \tag{2}$$

whose state space is the first quadrant $\mathbb{X} = \{(S, I) \in \mathbb{R}^2_+ : S > 0, I \geq 0\}$.

On the other hand, many researches have shown that environmental fluctuation has a huge influence on the development of infectious diseases [9,10]. For human disease, the spread and outbreak of the infectious disease is inherently stochastic due to the unpredictability of person-to-person contacts [11] and population suffer from a continuous spectrum of perturbations [12]. Therefore, the variability and randomness of the environment are fed through to the state of the epidemic [13]. A more realistic way of modeling infectious diseases is stochastic differential equation (SDE) models in many cases [2,11–19].

To incorporate the effect of environmental fluctuations, we formulate a stochastic differential equation (SDE) model by introducing the term multiplicative noise into the growth equations of both the susceptible and the infected populations [20] and assume that the natural death rate μ will fluctuate around some average value due to continuous environment fluctuation. And we introduce randomness into the deterministic model (2) by perturbing μ with $\mu - \sigma\zeta(t)$:

$$\begin{cases} \dfrac{dS}{dt} = \Lambda - (\mu - \sigma\zeta(t)) - (\beta_1 - \beta_2 f(I))SI + \gamma I, \\ \dfrac{dI}{dt} = (\beta_1 - \beta_2 f(I))SI - (\mu - \sigma\zeta(t) + \gamma)I, \end{cases} \tag{3}$$

where $\zeta(t)$ is a Gaussian white noise and characterized by:

$$\langle\zeta(t)\rangle = 0, \qquad \langle\zeta(t)\zeta(t')\rangle = \delta(t - t'),$$

where $\langle\cdot\rangle$ denotes ensemble average and $\delta(\cdot)$ is the Dirac-δ function. σ is a real constant which measures the intensity of environmental fluctuations. And we can rewrite model (3) into the form of stochastic differential equations as follows:

$$\begin{cases} dS_t = [\Lambda - \mu S_t - (\beta_1 - \beta_2 f(I))S_t I_t + \gamma I_t]dt + \sigma S_t dB_t, \\ dI_t = [(\beta_1 - \beta_2 f(I))S_t I_t - (\mu + \gamma)I_t]dt + \sigma I_t dB_t. \end{cases} \tag{4}$$

where B_t is the standard one-dimensional independent Wiener process defined over the complete probability space $\big(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \text{Prob}\big)$, and the relations between the white noise terms and Wiener process are defined by $dB_t = \zeta(t)dt$. And the state space of the SDE model (4) is $\mathbb{X}$, too.

The structure of this article is as follows: In Section 2, we give the analysis of the global disease dynamics of deterministic model (2). In Section 3, we analyze the disease dynamics of stochastic model (4). Numerical investigation and simulations