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Persistence of discrimination: Revisiting Axtell, Epstein and Young

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ABSTRACT

We reformulate an earlier model of the "Emergence of classes..." proposed by Axtell et al. (2001) using more elaborate cognitive processes allowing a statistical physics approach. The thorough analysis of the phase space and of the basins of attraction leads to a reconsideration of the previous social interpretations: our model predicts the reinforcement of discrimination biases and their long term stability rather than the emergence of classes.

1. Introduction

During the 90s social scientists introduced several thought provocative models of social phenomena, most often using numerical simulations (multi-agent simulations). These models have later been extended by methods and concepts derived from statistical physics such as Master Equations and Mean Field Approximation. A few examples include voters models and imitation processes [1] and the review of Castellano et al. [2] , El Farol and the minority game [3] and [4], diffusion of cultures [5,6]. Revisiting these models provided deeper insight, more precise results and even sometimes corrections.

The questions of the emergence and persistence of classes and discrimination received a lot of attention from social scientists, ethnographers and economists, see e.g. [7] and references within. A very inspiring model entitled "Emergence of Classes in a Multi-Agent Bargaining Model" was proposed by Axtell et al. [8]. We here propose to revisit their approach using a more elaborate model of agent cognition and to compare a mean field approach to our agent based simulation results.

2. The models

2.1. The original model of Axtell, Epstein and Young

Let us briefly recall the original hypotheses and the main results of Axtell et al. [8].

- Framework: pairs of agents play a bargaining game introduced by Nas, Jr. [9] and Young [10]. During sessions of the game, each agent can, independently of his opponent, request one among three demands: L(ow) demand 30 perc. of a pie, M(edium) 50 perc. and H(igh) 70 perc. As a result, the two agents get at the end of the session what they demanded when the sum of both demands is less than the 100 perc. total; otherwise they do not get anything. The

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Table 1
Payoff matrix of the Nash demand game. The first column represents the move of the first player L, M or H. The first row represents the move of her opponent. The figures in the matrix represent the payoff obtained by the first player.

	L	M	H
L	0.3	0.3	0.3
M	0.5	0.5	0
H	0.7	0	0

corresponding payoff matrix is written Table 1. At each step a random pair of agents is selected to play the bargaining game. The iterated game is played for a large number of sessions, much larger that the total number of agents which could then learn from their experience how to improve their performance.
– Learning and memory: Agents keep records of the previous demands of their opponents, e.g. for 10 previous moves.
– Choosing the next move: at each time step, pairs of agent are randomly selected to play the bargaining game. They most often choose the move that optimises their expected payoff using the memory of previous encounters as a surrogate for the actual probability distribution of their opponent's next moves. With a small probability ϵ, e.g. 0.1, they choose randomly among L, M, H.

The main results obtained by Axtell et al. [8] from numerical simulations are:

– They observe different transient configurations which they interpret as "norms", e.g. the equity norm is observed when all agents play M. Because of the constant probability of random noise, the system never stabilises on an attractor, even in the sense of Statistical Physics. The duration of the transients increases exponentially with the memory size and $1/\epsilon$.
– Their most fascinating result is obtained when agents are divided into two populations with arbitrary tags say e.g. one red and one blue. When agents take into account tags for playing and memorising games (in other words when agents play separately two games, one intra-game against agents with the same tag and another inter-game against agents with a different tag) one observes configurations in the inter-game such that one population always play H while the other population plays L; they interpret such inequity norm as the emergence of classes, the H playing population being the upper class.

Equivalent results are obtained when agents are connected via a social network as observed by Poza et al. [11] on a square lattice as opposed to the full connection structure used by Axtell et al. [8]. For some instances, domains with different norms occupy different parts of the lattice. Otherwise, one single domain of agents playing the same norm covers the entire lattice, depending upon the initial conditions.

From now on, we follow a plan starting with the exposition of our own model (Section 2.2). The use of a mean field approximation allows to simply describe the attractors of the dynamics and the different dynamical regimes (Section 3). These results are then compared with those obtained by direct agent based simulations (Section 4), including a thorough survey of the attraction basins. We further proceed with the analysis of the two tagged populations version (Section 5). The discussion compares our results to those of previous models and to magnetic systems. A short conclusion stresses the difference in interpretation of the models in terms of social phenomena (Section 6).

2.2. The moving average and Boltzmann choice cognitive model

We start from the same bargaining game as [8] with a payoff matrix written in Table 1, but using different coding of past experience (moving average of past profits) and choice function (Boltzmann function).

The present model is derived from standard models of reinforcement learning in cognitive science, see for instance [12].

Rather than memorising a full sequence of previous games, agents update 3 "preference coefficients" J_j for each possible move j, based on a moving average of the profits they made in the past when playing j. J_1 is the preference coefficient for playing H, J_2 for M and J_3 for L. The updating process following time interval τ after a transaction is:

$$J_j(t+\tau) = (1-\gamma)\cdot J_j(t) + \pi_j(t), \qquad \forall j, \tag{1}$$

The decrease term in $1-\gamma$ corresponds to discounting the importance of past transactions, which makes sense in an environment varying with the choices of the other players. $\pi_j(t)$ is the actual profit made during the chosen transaction j; the 2 other $J_{j'}$ corresponding to the 2 other choices j' are simply decreased.

These preference coefficients are then used to choose the next move in the bargaining game. Agents face an exploitation/exploration dilemma: they can decide to exploit the information they earlier gathered by choosing the move with the highest preference coefficient or check possible evolutions of profits by trying randomly other moves. Rather than using a constant rate of random exploration ϵ as in [8], the probability of choosing demand j is based on the logit function:

$$P_j = \frac{\exp(\beta J_j)}{\sum_j \exp(\beta J_j)}, \qquad \forall j, \tag{2}$$