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Time-localized wavelet multiple regression and correlation

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HIGHLIGHTS

- This paper introduces the local multiscale regression analysis along time.
- It presents the WMLC as a new tool in the analysis of evolving correlation structures.
- For Euro stock markets the WLMC shows a sharp divide at the quarterly timescale.
- WLMC analysis reveals perfect integration of Euro stock markets at long timescales.
- It also finds evidence of contagion at short scales during 2007–12 financial crises.

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ABSTRACT

This paper extends wavelet methodology to handle comovement dynamics of multivariate time series via moving weighted regression on wavelet coefficients. The concept of wavelet local multiple correlation is used to produce one single set of multiscale correlations along time, in contrast with the large number of wavelet correlation maps that need to be compared when using standard pairwise wavelet correlations with rolling windows. Also, the spectral properties of weight functions are investigated and it is argued that some common time windows, such as the usual rectangular rolling window, are not satisfactory on these grounds.

The method is illustrated with a multiscale analysis of the comovements of Eurozone stock markets during this century. It is shown how the evolution of the correlation structure in these markets has been far from homogeneous both along time and across timescales featuring an acute divide across timescales at about the quarterly scale. At longer scales, evidence from the long-term correlation structure can be interpreted as stable perfect integration among Euro stock markets. On the other hand, at intramonth and intraweek scales, the short-term correlation structure has been clearly evolving along time, experiencing a sharp increase during financial crises which may be interpreted as evidence of financial 'contagion'.

1. Introduction

One important aspect in the analysis of economic and financial time series is the study of the degree and behavior of their comovements at different periods and frequencies. Whilst traditional time series analysis (TSA) is mostly based on cross-correlation functions in the time domain, that is, on an observation-by-observation basis, Fourier analysis allows for the visualization of the data in the frequency domain, that is, on a frequency-by-frequency basis. In other words, traditional TSA provides full time resolution of such relationships but does not expose frequency information while Fourier analysis has

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full frequency resolution but does not preserve information in time. These types of analysis implicitly assume stationarity, possibly after differences, as the main characteristic of the time series under study [1, p. 435].

The more recent wavelet analysis emerges as a compromise between both approaches, with partial resolution in both time and frequency domains. Thus, the continuous wavelet transform (CWT) allows for a visualization of the spectral features of the time series and their comovements but as a function of both time and scale (frequency), separating their different periodic components as they evolve over time [2,3].

The CWT is highly redundant on both time and scale dimensions, the latter not being a desirable feature for regression/correlation decomposition over timescales. On the other hand, the discrete wavelet transform (DWT) selects a minimal subsample of time–frequency values from the CWT without losing any information present in the original data. This is an important feature for some engineering applications like signal and image compression, but for most economic applications time redundancy is somehow desirable as long as it allows for data features to be properly aligned and compared across all scales/frequencies. In this sense, the maximal overlap discrete wavelet transform (MODWT) is the most popular wavelet transform as it is redundant in the time dimension but non-redundant in the scale/frequency dimension. MODWT is known to have several important advantages, including energy preservation which is particularly important for the main objective of this paper ([2,4]; [5, p. 135]).

MODWT based wavelet multiple correlations (WMC) and cross-correlations (WMCC) were introduced by Fernández-Macho [6]. These statistics distribute among the different timescales the overall statistical relationship that might exist between several time series and they have received some attention in the recent literature [see *e.g.* [3,7–15], among others]. However, as with standard bivariate wavelet correlations, it is implicitly assumed that the time series follow difference-stationary processes and, therefore, that there exists a sufficiently long wavelet filter that eliminates this type of nonstationarity from the data. In consequence, only one single global correlation per scale needs be produced.

This notwithstanding, with economic and financial time series, such data features are probably non-stationary in nature and a regression/correlation analysis must be able to handle and visualize a changing structure that evolves along time. Therefore, the present paper generalizes the previous global WMC to a local multiple regression framework where comovement dynamics across the different scales/frequencies can be analyzed along time by using weighted or windowed wavelet coefficients.

The proposed method is justified on several grounds. multivar vs bivar First of all, the alternative of combining standard bivariate wavelet correlation analysis with rolling time windows needs to calculate, plot and compare a large number of wavelet correlation graphs that now would require an additional time dimension [7,16–18, *etc.*]. More specifically, with n time series, a total of $n(n-1)/2$ wavelet correlation maps would be required each of dimension $J \times T$, where $J = \lfloor \log_2(T/(L-1)+1) \rfloor$ is the order of the wavelet transform, L is the wavelet filter length and T is the time series length.[1] Not surprisingly, when dealing with all possible pairwise comparisons in a multiscale context, the analyst may end up with a vast amount of potentially conflicting information that can be very difficult to process and even lead to the typical experimentwise error rate inflation and the spurious detection of correlations at some wavelet scales [6]. In contrast, the proposed method based on local multiple regression, consists in one single set of multiscale relationships which can be expressed in a single scale-time correlation map. This is not only easier to handle and interpret but also may provide a better insight of the overall statistical comovement dynamics within the multivariate time series under scrutiny. number of feasible scales Also, in the proposed method the number of feasible scales remains the same as in global wavelet analysis and does not depend on the length of the time window. Therefore, long wavelet filters and windows of long length or even with infinite support like the Gaussian weight function can be used. which window (weight function)? Finally, as discussed in Section 4, the spectral properties of the weight function need to be taken into account as, for example, the usual rectangular rolling window will not be satisfactory on these grounds.

All this will be illustrated with the application of the proposed Wavelet Local Multiple Correlation (WLMC) in the multiscale analysis of daily returns obtained from a set of eleven Eurozone stock markets during the 17-year period of the present century during which several financial and debt crises have occurred. In this relation, we may point out that correlation among European stock markets is a common measure of market integration in the economic and financial literature [see, *e.g.*, [19–23], and others]. However, these studies do not usually take into account the fact that stock markets involve heterogeneous agents that make decisions over different time horizons ([5, p. 10], [24]), or that such comovement structure across different timescales may be evolving along time. On the other hand, the relatively large but not uncommon number of markets to be analyzed will render, as already mentioned, pairwise multiscale comparisons pointless in practice, which is the reason why this type of market analysis may find useful the wavelet local multiple regression tools proposed here.

The paper is organized as follows. Sections 2 and 3 set up the framework for the proposed wavelet local multiple regression tools and extend such framework with the definition of the WLMC considering the decomposition of the time series correlation structure across different timescales. Sample estimators of these quantities and their approximate confidence intervals based on their large sample theory are also provided for estimation and testing purposes. Section 4 discusses the spectral properties of common weight functions in local regression that can be used in practice and Section 5 presents some examples: the first two with simulated changes along time in the correlation structure across different timescales that serve to illustrate the validity of the proposed statistics and, finally, a case study that shows the results of an empirical application using Eurozone stock markets. Section 6 summarizes the main conclusions.

[1] For example, the empirical analysis in Section 5.3 would need to handle a total of 55 wavelet correlation maps of dimension 9×4542 each.