Physica A 465 (2017) 552–561

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Comparisons of different witnesses of non-Markovianity

Wei Zuo, Xiao-Qing Qian, Xian-Ting Liang *

Department of Physics and Institute of Optics, Ningbo University, Ningbo 315211, China

HIGHLIGHTS

- Two kinds of witnesses of non-Markovianity have been calculated and compared.
- Witnesses based on the completely positive and local completely positive maps, are equivalent respectively.
- Non-Markovianity decreases with the increase of environmental noises, but is not affected by the choice of the initial states.

ARTICLE INFO

Article history:
Received 31 March 2016
Received in revised form 30 July 2016
Available online 26 August 2016

Keywords:
Non-Markovianity
Witness
Quantum information

ABSTRACT

In this paper, the evolutions of two kinds of witnesses of the non-Markovianity and their rates of changes with time are investigated and compared. Four definitions, the trace distance, fidelity, quantum relative entropy, and quantum Fisher information are used for the first kind of witnesses which are based on the completely positive maps (CPM). Three definitions, the quantum entanglement, quantum mutual information, and quantum discord are used for the second kind of witnesses, and they are based on the local completely positive maps (LCPM). An open two-level quantum system model and a numerically quantum dissipative dynamics method, hierarchy equation of motion (HEM) are used in the investigations. It is shown that the evolutions of the witnesses and their rates of the changes calculated with different definitions clearly show the characteristics of the non-Markovianity and they are in agreement with each other.

1. Introduction

Markovianity, named after the Russian mathematician A. Markov [1], is a type of classical process that does not have memory of the past. It has been widely investigated in many disciplines in the past few decades [2,3]. For a quantum system, it is difficult in general to define the quantum Markovian process. However, in the sense of divisibility, if its dynamical evolution can be described by some family of trace-preserving linear maps, namely, $\rho(t_1) = \varepsilon(t_1, t_0)\rho(t_0)$, and, for every t_2 and t_1, $\varepsilon(t_2, t_1)$ is a complete positive map and fulfills the composition law as

$$\varepsilon(t_3, t_1) = \varepsilon(t_3, t_2)\varepsilon(t_2, t_1), \quad t_3 > t_2 > t_1, \tag{1}$$

it is Markovian [1]. Thus, some dynamical approaches, for example, the Redfield and Lindblad forms of the master equations, describe this kind of process. It is easy to discern from Eq. (1) that the evolution of a closed quantum system is Markovian. However, the practical dynamics of an open quantum system is essentially non-Markovian.

For the Markovian processes, the correlation time between the system and environment is considered to be infinitesimally small so that the memory effects of the dynamical map can be neglected, so it will lead to a monotonic flow of the information from system to environment. In contrast, for non-Markovian processes, the memory effects of the environment cannot be ignored, so it always results in a rise to the back flow of information from environment to the system [4].

* Corresponding author.
E-mail address: liangxianting@nbu.edu.cn (X.-T. Liang).

http://dx.doi.org/10.1016/j.physa.2016.08.058
0378-4371/

Theoretically, many non-Markovian dynamical approaches have been developed in recent years [5–10]. To quantify the non-Markovianity, some so-called measures of the non-Markovianity, such as geometric measure, trace norm, decay rate measure, and hierarchical k-divisibility degree, have been introduced. The main drawback of quantifying the non-Markovianity with these measures is the difficulty in optimizing the process, which makes these measures rather impractical. It is much easier to measure the non-Markovianity via witnesses. A witness of the non-Markovianity is a quantity that vanishes for all Markovian dynamics, but it may also vanish for some non-Markovian ones. Thus, when a witness of the non-Markovianity gives a nonzero value, we are sure that the dynamics is non-Markovian. Consequently, investigations of the non-Markovianity by using witnesses become much easier than by using the measures, because we do not have to optimize the processes. The witnesses of the non-Markovianity presented in the literature are classified into two kinds according to two guiding principles. One kind of the witnesses is based on monotonic quantities under completely positive maps (CPM), and the other based on monotonic quantities under local completely positive maps (LCPM). The former includes trace distance, fidelity, quantum relative entropy, quantum Fisher information, capacity measure, Bloch volume measure etc., and the latter includes entanglement, quantum mutual information, quantum discord and so on.

It is interested that what are the differences between these witnesses, or whether they are equivalent in the descriptions of the non-Markovianity. In this paper, we shall calculate some witnesses by using several different definitions and the same model. The quantum dissipative dynamical method, the hierarchy equation of motion (HEM) method [11], will be used. It will be shown that the witnesses calculated from the different definitions belonging to the two kinds of maps are exactly equivalent respectively for the descriptions of non-Markovianity of the investigated model.

2. Dynamical method

The Hamiltonian of an open quantum system is given by

$$H = H_s + \sum_k \left[\frac{p_k^2}{2m_k} + \frac{1}{2} m_k \omega_k^2 \left(x_k - \frac{c_k \sigma_z}{m_k \omega_k^2} \right)^2 \right], \tag{2}$$

where σ_z is a Pauli matrix, H_s represents the Hamiltonian of the system, and the environment described with the harmonic oscillators are coupled to the modes of the system. The dynamics of the system can be described with quantum master equations (QMEs) in Markovian approximation. This kind of equations, for example the Redfield and Lindblad forms are in general only suitable for weak system–bath interactions as memory effects of the bath are neglected. A direction for solving the quantum dissipative dynamics including non-Markovian effects is based on the path integral formalism [12,13]. Marki and co-workers have developed an exact numerical dynamics approach — quasi-adiabatic propagator path integral (QUAPI) method [14,5]. This is an efficient way of including memory effects via tensor products of element memory kernels. A different approach was proposed by Tanimura and co-workers, who introduced a hierarchical treatment of non-Markovian dynamics, in which the primary density operator is coupled to auxiliary density operators, describing the effects of successively higher order system–bath interaction [15,16]. These two kinds of non-Markovian dynamical methods can include dynamical memory effects, and are extensively used in the investigations of decoherence, disentanglement, energy transformation, and spectral analysis in quantum systems. Many other methods have also been proposed and used in last years [17,18]. In the following, we shall use the hierarchy equation of motion (HEM) to investigate the witnesses of non-Markovianity.

If we set the environmental density function with the Drude–Lorentz spectral form [19,11]:

$$J(\omega) = \frac{\hbar\eta}{\pi\omega_0} \frac{\gamma^2 \omega}{\omega^2 + \gamma^2}. \tag{3}$$

Here, η is related to the system–bath coupling strength, while γ represents the width of the spectral distribution of the bath modes and it is related to the correlation time of the noise induced by the bath, $t_c = 1/\gamma$. In this study, we use the following hierarchy of equations in operator form [11,20]:

$$\begin{aligned} \frac{\partial}{\partial t}\rho^{(n)}_{j_1,j_2,\ldots,j_K}(t) = {} & -\frac{i}{\hbar} H_s^{\times} \rho^{(n)}_{j_1,j_2,\ldots,j_K}(t) - \left(n\gamma + \sum_{k=1}^{K} j_k \nu_k - \Xi(t) \right) \rho^{(n)}_{j_1,j_2,\ldots,j_K}(t) + n\Theta(t)\rho^{(n-1)}_{j_1,j_2,\ldots,j_K}(t) \\ & + \sum_{k=1}^{K} j_k \nu_k \Psi_k(t) \rho^{(n)}_{j_1,\ldots,j_k-1,\ldots,j_K}(t) + V^{\times} \rho^{(n+1)}_{j_1,j_2,\ldots,j_K}(t) + \sum_{k=1}^{K} V^{\times} \rho^{(n)}_{j_1,\ldots,j_k+1,\ldots,j_K}(t). \end{aligned} \tag{4}$$

Here, $\Theta(t)$, $\Psi_k(t)$ and $\Xi(t)$ are the operator forms of

$$\begin{aligned} \Theta(t) &= \frac{\eta\gamma^2}{2\omega_0} \left[iV^{\circ}(t) - \cot\left(\frac{\beta\hbar\gamma}{2} \right) V^{\times} \right], \\ \Psi_k(t) &= \frac{\eta\gamma^2}{\beta\hbar\omega_0} \frac{2}{\gamma^2 - \upsilon_k^2 V^{\times}(t)}, \\ \Xi(t) &= \frac{i\eta}{\beta\hbar\omega_0} \frac{2\gamma^2}{\upsilon_k^2 - \gamma^2} V^{\times}(t) V^{\times}(t). \end{aligned} \tag{5}$$