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Q1 # A kinematic wave model in Lagrangian coordinates incorporating capacity drop: Application to homogeneous road stretches and discontinuities

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HIGHLIGHTS

- A methodology of incorporating the capacity drop into the kinematic wave model.
- A kinematic wave model solved in Lagrangian coordinates.
- Applications to homogeneous road stretches and discontinuities.
- The capacity drop reproduced by the model is related to the congestion characteristic.

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ABSTRACT

On freeways, congestion always leads to capacity drop. This means the queue discharge rate is lower than the pre-queue capacity. Our recent research findings indicate that the queue discharge rate increases with the speed in congestion, that is the capacity drop is strongly correlated with the congestion state. Incorporating this varying capacity drop into a kinematic wave model is essential for assessing consequences of control strategies. However, to the best of authors' knowledge, no such a model exists. This paper fills the research gap by presenting a Lagrangian kinematic wave model. "Lagrangian" denotes that the new model is solved in Lagrangian coordinates. The new model can give capacity drops accompanying both of stop-and-go waves (on homogeneous freeway section) and standing queues (at nodes) in a network. The new model can be applied in a network operation. In this Lagrangian kinematic wave model, the queue discharge rate (or the capacity drop) is a function of vehicular speed in traffic jams. Four case studies on links as well as at lane-drop and on-ramp nodes show that the Lagrangian kinematic wave model can give capacity drops well, consistent with empirical observations.

1. Introduction

Researchers have devoted much effort to finding solutions to congestion problems. The dynamic traffic operation is possibly one of the most cost-effective strategies. Empirical research provides theoretical bases for promising freeway operations. One of those most crucial empirical findings is the capacity drop phenomenon, which is confirmed empirically in Refs. [1,2].

Once congestion occurs on a freeway, the flow downstream of a traffic jam is lower than the pre-queue capacity. This phenomenon is called the capacity drop. The maximum flow downstream of the jam is called the queue discharge rate.

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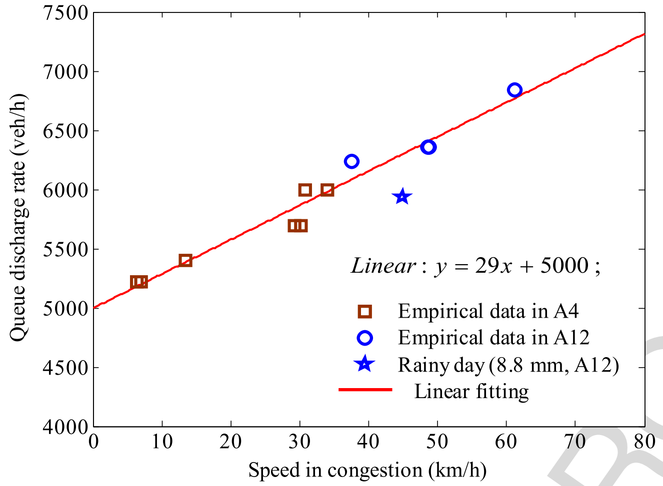

Fig. 1. Empirical data showing a relation between the speed in congestion and the queue discharge rate [4].

We divide congestion into two categories: stop-and-go waves which propagate upstream with both congestion fronts, and standing queues, whose heads are fixed at bottlenecks. In this research, this division in congestion is due to the difference in modeling traffic flow on links and nodes.

Incorporating the capacity drop into traffic flow models is crucial for the evaluation of dynamic traffic operations, as argued in Ref. [3]. Traffic delays strongly depend on the capacity drop. A model which does not include the capacity drop cannot show benefits obtained from some traffic management strategies (e.g., on-ramp metering strategy). Following earlier works on the empirical observations on the capacity drop, we argue that a model that well incorporates the capacity drop should (i) give a queue discharge rate reduction from the pre-queue capacity and (ii) be able to produce a relation between the queue discharge rate and the speed in congestion, see Fig. 1. The relation is presented in Ref. [4] with loop detector data collected on freeway A4 and A12 in the Netherlands. Yuan et al. [5] argue that this relation is a result of intra-driver variation mechanism. Giving those two phenomena are relevant for traffic flow models because the accuracy of the reproduced capacity drop influences the values of control variables and even the strategy performances.

The aim of this paper is to incorporate the capacity drop into a macroscopic traffic flow model for traffic operations. A macroscopic traffic flow model is preferred due to its analytical properties. For instance, solutions to macroscopic models are frequently described as closed-form expressions, and are deterministic [6]. Nowadays researchers frequently use second order models to simulate the traffic state evolution on freeways, such as in Refs. [7,8]. The second order model has the ability of giving capacity drop. However, the improved accuracy of the second order model comes at a cost of complexity (e.g., more model parameters) and assumptions. And the second order model can result in negative flow and fast characteristic wave speed. For details, we refer to Ref. [9]. By contrast, these issues do not occur in a first order model. Hence, we argue that it is preferred to incorporate the capacity drop into the kinematic wave model.

Earlier works have tried some approaches to capture the capacity drop in the kinematic wave model, e.g., a discontinuous fundamental diagram [10]. However, firstly the discontinuous fundamental diagram will result in infinite shock wave speed and characteristic wave speed, as argued in Ref. [11]. Secondly, the discontinuity obtained from empirical data could be a result of non-stationary traffic [12]. Therefore, we believe that it is best to use a continuous fundamental diagram rather than a discontinuous one.

Some efforts have been made, with continuous fundamental diagram, to capture capacity drop in the kinematic wave model. Generally, those efforts are to revise the demand or supply function of cells in the vicinity of congestion. Maria et al. [13] reduce the supply of the immediate downstream cell of congestion by introducing an auxiliary variable illustrating the maximum flow for each cell. Srivastava et al. [14] assume two values of capacity. If the cell is congested during the last time step, the lower-value capacity is used to restrict the supply; or the other higher value is the maximum supply. Alvarez-Icaza and Islas [15] apply a hysteresis cycle to decide the supply function which is related to the wave speed. But in these works, the capacity drop magnitude is fixed, independent from the congestion states. They cannot reproduce the relation between the queue discharge rate and the congested state.

Some works take account of bounded acceleration effects for giving capacity drop. Literature [16,17] show that when the density exceeds the critical density, the bounded acceleration effects modify the constant demand to a negative function of density. Combining the LWR-BA model, a bounded acceleration Lighthill–Whitham–Richards (LWR) model by Refs. [16,17], and a node model proposed by Lebacque [18], can give capacity drop as a result of bounded traffic acceleration in the LWR framework [19,20]. However, this LWR-BA model cannot give queue discharge rate reductions of stop-and-go waves on homogeneous road sections where the node model should be absent.

Some other works consider lane changing to give capacity drop. Muralidharan and Horowitz [21] decrease the supply function at merge cells with a weaving parameter. The weaving parameter illustrates that the merging behavior from the

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