Physica A 465 (2017) 621–627

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Synchronization effect for uncertain quantum networks

Wenlin Li, Tesfay Gebremariam, Chong Li *, Heshan Song

School of Physics and Optoelectronic Engineering, Dalian University of Technology, 116024, China

HIGHLIGHTS

- A novel technique is proposed to investigate the synchronization effect for uncertain networks with quantum chaotic behaviors.
- A special function is designed to construct Lyapunov function of network.
- The identification laws of uncertain parameters in network are designed.

ARTICLE INFO

Article history:
Received 28 June 2016
Received in revised form 8 August 2016
Available online 29 August 2016

Keywords:
Synchronization
Uncertain network
Identification
Lyapunov theorem

ABSTRACT

We propose a novel technique for investigating the synchronization effect for uncertain networks with quantum chaotic behaviors in this paper. Through designing a special function to construct Lyapunov function of network and the adaptive laws of uncertain parameters, the synchronization between the uncertain network and the synchronization target can be realized, and the uncertain parameters in state equations of the network nodes are perfectly identified. All the theoretical results are verified by numerical simulations to demonstrate the effectiveness of the proposed synchronization technique.

1. Introduction

In recent years, complex networks have attracted increasing attention for its large size and complex topology. In complex networks, the most typical collective behavior is the synchronization phenomena. Up to now, the synchronization of complex networks has become hot topic and exhibited its application potential in various fields, such as secure communication, automatic control, WWW, Internet, and so on [1–6].

As we know now, many effective synchronization techniques including Master stability function method [7], adaptive control [8,9], pinning technique [10,11], impulsive control [12] have been proposed to drive the network to achieve synchronization. Among them, some typical work such as, Selivanov et al. finish adaptive synchronization in delay-coupled networks of Stuart–Landau oscillators [13]. Lü et al. research lag projective synchronization of a class of complex network constituted nodes with chaotic behavior [14]. Rakkiyappan et al.'s study examines the problem of synchronization for singular complex dynamical networks with Markovian jumping parameters and two additive time-varying delay components [15]. Murguia et al. research the problem of controlled synchronization in networks of nonlinear systems interconnected through diffusive time-delayed dynamic couplings [16].

It is worth noting that the above-mentioned researches on synchronization of networks mainly focused on the phenomenon that all nodes in a network achieve a coherent behavior, which was called inner synchronization. In reality, synchronization among two or more networks which is called outer synchronization always does exist. There are so many examples that can be taken to illustrate the phenomenon of outer synchronization, for instance, the infectious disease

* Corresponding author.
E-mail address: lichong@dlut.edu.cn (C. Li).

http://dx.doi.org/10.1016/j.physa.2016.08.064
0378-4371/

spreads between different communities, the avian influenza spreads among domestic and wild birds, and the different species development in balance. Thus, the outer synchronization has attracted more and more attention [17–20]. On the other hand, most of the results in the literature on the synchronization of complex networks deal with the networks with certain parameters. This assumption cannot be satisfied in many real situations whereby it is not easy to determine all the network parameters in advance. Therefore, research on the synchronization of uncertain networks has more practical value [21–23].

Based on above discussions, a novel technique is proposed in our work to investigate the synchronization effect for uncertain networks with quantum chaotic behaviors. Through designing a special function to construct Lyapunov function of network and the adaptive laws of uncertain parameters, the synchronization between the uncertain network and the synchronization target can be realized, and the uncertain parameters in state equations of the network nodes are perfectly identified. All the theoretical results are verified by numerical simulations to demonstrate the effectiveness of the proposed synchronization technique.

2. The synchronization mechanism of the uncertain network

In this section, we focus on the synchronization mechanism of the uncertain network. Consider a dynamical network consisting of M nodes, with each node being an n-dimensional dynamical system. The state equations of the network can be described by

$$\begin{aligned}\dot{x}_i(t) &= F(x_i, \alpha_i) + \sigma_i \sum_{j=1}^{M} b_{ij} x_j(t) + u_i(t) \\ &= f(x_i) + g(x_i)\alpha_i + \sigma_i \sum_{j=1}^{M} b_{ij} x_j(t) + u_i(t) \quad (i = 1, 2, \ldots, M)\end{aligned} \tag{1}$$

where $x_i(t)$ are the state variables of the network node, σ_i is the coupling strength between network nodes, and $u_i(t)$ denotes the control input of network. b_{ij} represents the matrix element of coupling matrix, which describes the topology structure of network. It is defined as follows $\sum_{j=1}^{N} b_{ij} = 0$. α_i is uncertain parameter of the network.

We assume that the synchronization target is

$$\dot{x}_d(t) = F(x_d, \alpha). \tag{2}$$

The error of state variables belonging respectively to the synchronization target and the network node is defined as

$$e_i(t) = x_i(t) - x_d(t) \quad (i = 1, 2, \ldots, M). \tag{3}$$

So the error equation is

$$\dot{e}_i(t) = f(x_i) + g(x_i)\alpha_i + \sigma_i \sum_{j=1}^{M} b_{ij} x_j(t) + u_i(t) - F(x_d, \alpha). \tag{4}$$

We choose a special function as

$$Q_i(t) = \left(\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_i\right) \varphi_i(t) \tag{5}$$

where $\varphi_i(t)$ is a function given by

$$\varphi_i(t) = \int_0^t e_i(\theta)\mathrm{d}\theta \tag{6}$$

and λ_i is an arbitrary positive coefficient.

We construct Lyapunov function of the network as

$$V = \frac{1}{2} \sum_{i=1}^{M} Q_i(t)^T Q_i(t) + \frac{1}{2} \sum_{i=1}^{M} \xi_i (\hat{\alpha}_i - \alpha_i)^T (\hat{\alpha}_i - \alpha_i) \tag{7}$$

where $\hat{\alpha}_i$ stands for the identification of the uncertain parameter in the network and ξ_i is an adjustment parameter.

Consider Lipschitz condition, that is, for the real number $l_i > 0$, the following relationships exist

$$\left|F(x_i, \hat{\alpha}_i) - F(x_d, \alpha)\right| \le l_i \left|x_i(t) - x_d(t)\right|. \tag{8}$$