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The role of mixed strategies in spatial evolutionary games

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ABSTRACT

We study three-strategy evolutionary games on a square lattice when the third strategy is a mixed strategy of the first and second ones. It is shown that the resultant three-strategy game is a potential game as well as its two-strategy version. Evaluating the potential we derive a phase diagram on a two-dimensional plane of rescaled payoff parameters that is valid in the zero noise limit of the logit dynamical rule. The mixed strategy is missing in this phase diagram. The effects of two different dynamical rules are analyzed by Monte Carlo simulations and the results of imitation dynamics indicate the dominance of the mixed strategy within the region of the hawk–dove game where it is an evolutionarily stable strategy. The effects and consequences of the different dynamical rules on the final stationary states and phase transitions are discussed.

1. Introduction

In most of the multi-agent evolutionary games [1–3] the interactions among the equivalent players are described by a suitable sum of symmetric two-person games characterized by a uniform $n \times n$ payoff matrix $\mathbf{A}$ [4] where n is the number of strategies. In the spatial evolutionary games the players are distributed on a lattice and their individual income comes from games played with their nearest neighbors. In these models the players are allowed to modify their own strategy by following a dynamical rule that may be based on the deterministic [5] or stochastic [6] imitation of a better (or the best) neighbor, or favoring the selection of a better strategy with assuming fixed strategies in the neighborhood [7–11], *etc.*. In the last years the models with a logit dynamical rule are investigated more and more frequently in the literature of physics (for a recent review see Ref. [12]) because this dynamical rule drives the systems into a Boltzmann distribution if the pair interactions are defined by potential games [13–16]. The similarity between the Ising type models [17,18] and some two-strategy evolutionary games was reported previously by many authors [19–24].

In normal games the players have a finite number of pure strategies and each player has a quantified payoff function dependent on her own strategy and also on the strategies chosen by the co-players. Evidently, these games involve different types of interactions [25,12] including situations when conflicts can occur between the interest of selfish (rational) players who wish to maximize their own payoff irrespective of the others [26,16]. For the potential games we can derive a potential, as a function of strategy profiles, that comprises the individual interest of the active player when only unilateral strategy changes are allowed in the consecutive steps during the evolutionary process.

The potential games have some curious features that make them attractive when drawing parallels between the evolutionary games and models of statistical physics. First we emphasize that the potential games have one or more pure

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Nash equilibria from which the unilateral deviation is not beneficial for the active player. If the potential has a single maximum then the corresponding strategy profile can be considered as a preferred pure Nash equilibrium to which the system evolves in the low noise limit for the logit rule. The concept of potential and preferred Nash equilibrium are analogous to the negative potential energy (Hamiltonian) and ground state of a many particle system.

Now we will study in parallel two- and three-strategy evolutionary games with players located on a square lattice and playing games with their nearest neighbors. For the definition of the symmetric two-strategy games we use the notation of social dilemmas with rescaled payoffs characterizing the interactions with only two parameters. All the latter evolutionary games are potential games which can be mapped onto a kinetic Ising model for the application of the logit rule. At the same time the two-strategy symmetric games have an additional mixed Nash equilibrium in certain regions of the two-dimensional parameter space. The mentioned mixed Nash equilibrium becomes relevant for the hawk–dove games where it is an evolutionarily stable strategy [27] and plays a crucial role in biological systems described by population dynamics [28]. Some additional features of the mixed strategies are considered in several previous papers [29–34]. Now the role of mixed strategies is analyzed by discussing a three-strategy game that is derived from the two-strategy games with introducing one of the mixed strategies as a third pure strategy. It is found that the resultant three-strategy game is also a potential game and the third (mixed) strategy plays a minor role in the evolutionary game at low noises if a logit rule controls the evolution of strategy distribution. On the contrary, the dominance of mixed strategies was indicated by the Monte Carlo simulations if the dynamics is governed by random sequential imitations of a better neighbor via stochastic pairwise comparison of payoffs.

In the next section we define the formalism and models, evaluate the potentials and phase diagrams in the zero noise limit, and discuss the general features of the mixed strategies. The numerical analysis of the spatial evolutionary games is focused on the determination of the strategy frequencies when varying the noise level for the logit rule and also on the evaluation of phase diagram for imitation at a low noise level. Additionally we will illustrate the appearance of consecutive phase transitions when one of the payoff parameters is tuned at a fixed (low) noise. The results are detailed in Section 3 by considering separately the consequences of logit rule and imitation. Finally we survey the main messages.

2. Formalism, models, and general features

2.1. Two-strategy games

In the present spatial evolutionary games the equivalent players are located at the sites x of a square lattice. For the two-strategy models each player chooses one of her two options, called pure strategies, denoted traditionally by two-dimensional unit vectors as

$$\mathbf{s}_x = D = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad C = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

where D and C refer to defection and cooperation for the prisoner's dilemma games. Using these strategies the players play games with their four nearest neighbors. The income (u_x and u_y) of two neighboring players (at sites x and y) are given by products,

$$u_x = \mathbf{s}_x \cdot \mathbf{A}\mathbf{s}_y \quad \text{and} \quad u_y = \mathbf{s}_y \cdot \mathbf{A}\mathbf{s}_x \tag{2}$$

where the element A_{ij} of the payoff matrix $\mathbf{A}$ defines the payoff for the first player if she chooses her ith strategy whereas the co-player selected the jth strategy ($i, j = 1, 2$ referring to D and C).

Using the notation of social dilemmas [26] with rescaled payoffs the payoff matrix $\mathbf{A}$ is described by two parameters as

$$\mathbf{A} = \begin{pmatrix} 0 & T \\ S & 1 \end{pmatrix} \tag{3}$$

where T defines the temptation to choose defection, S is the sucker's payoff, $R = 1$ is the reward for mutual cooperation, and $P = 0$ refers to punishment for mutual defections within the region of prisoner's dilemma where $T > 1$ $S < 0$. In the T–S plane of parameters four types of games are distinguished in the left plot of Fig. 1 by illustrating the corresponding flow graphs. In these flow graphs boxes with labels indicate strategy profiles and the directed edges connect strategy pairs where only one of the players modifies her strategy. The arrows point to the preferred strategy of the active player. Here the nodes without outgoing edges are Nash equilibria. This plot illustrates that in the regions of the prisoner's dilemma (PD) and Harmony (H) games there is only one pure Nash equilibrium and two pure Nash equilibria exist for the hawk–dove (HD) and stag hunt (SH) games. The boundaries ($S = 0$ and $T = 1$) between the different types of games are denoted by dashed–dotted lines.

The sum of the payoff variations is zero along the single four-edge loop for all the flow graphs of the symmetric two-player two-strategy games. Consequently, these games are potential games [13,15] and we can derive a symmetric potential matrix [12]

$$\mathbf{V} = \begin{pmatrix} 0 & S \\ S & 1 + S - T \end{pmatrix} \tag{4}$$