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Long memory behavior of returns after intraday financial jumps

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HIGHLIGHTS

- Charaterization and time dynamics of returns after intraday financial jumps.
- Long memory with power-law distribution of return after jump.
- Power-law distribution ($\sim 1/r^{1+\mu}$) of jump tail with the exponent $1 < \mu < 2$.

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ABSTRACT

In this paper, characterization of intraday financial jumps and time dynamics of returns after jumps is investigated, and will be analytically and empirically shown that intraday jumps are power-law distributed with the exponent $1 < \mu < 2$; in addition, returns after jumps show long-memory behavior. In the theory of finance, it is important to be able to distinguish between jumps and continuous sample path price movements, and this can be achieved by introducing a statistical test via calculating sums of products of returns over small period of time. In the case of having jump, the null hypothesis for normality test is rejected; this is based on the idea that returns are composed of mixture of normally-distributed and power-law distributed data ($\sim 1/r^{1+\mu}$). Probability of rejection of null hypothesis is a function of μ, which is equal to one for $1 < \mu < 2$ within large intraday sample size M. To test this idea empirically, we downloaded S&P500 index data for both periods of 1997–1998 and 2014–2015, and showed that the Complementary Cumulative Distribution Function of jump return is power-law distributed with the exponent $1 < \mu < 2$. There are far more jumps in 1997–1998 as compared to 2015–2016; and it represents a power law exponent in 2015–2016 greater than one in 1997–1998. Assuming that i.i.d returns generally follow Poisson distribution, if the jump is a causal factor, high returns after jumps are the effect; we show that returns caused by jump decay as power-law distribution. To test this idea empirically, we average over the time dynamics of all days; therefore the superposed time dynamics after jump represent a power-law, which indicates that there is a long memory with a power-law distribution of return after jump.

1. Introduction

There are always fluctuations in the price due to general economic factors such as supply and demand, changes in economic outlook (endogenous factors). These factors cause small movements in the price and are modeled by a geometric

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Brownian motion with a constant drift term. In turn, jumps associate with the arrival of important information into the market (exogenous factors) that will have an abnormal effect on the price. Therefore, it is important to distinguish between jumps and continuous price movements as it has implications for risk management and asset allocation. As market distribution has heavier tails than a normal distribution, a model should allow large fluctuations such as crashes or jumps. Additionally, the market distribution is generally negatively skewed as downward returns and tail events are usually larger than upward ones.

In financial economics, one way to obtain variation of asset prices is by calculating the quadratic variation of returns (sum of discrete squared returns), named Realized quadratic variation. To find out more about this, see Barndorff Nielsen and Shephard [1,2], Andersen and Bollerslev [3], and Comte and Renault [4]. This was later developed as methodology for forecasting by Andersen et al. [5–7], and was generalized to Realized covariation by Barndorff-Nielsen and Shephard [8]. Later, a method was introduced by Barndorff-Nielsen and Shephard [9,10] introducing a quadratic variation (QV) and Bipower variation (BPV) methods to detect jumps. In this paper, we use similar method in order to split the variation into the continuous term and the jump term, and demonstrate briefly how one can use it to detect jumps. In fact, a null hypothesis of absence of jump is tested with an elaborated statistic formula, where under the null it is normally distributed, and in the case of having a jump, the null hypothesis for normality test is rejected. Via this approach, number of jumps and jump returns within financial intraday data are computed.

There is a dense econophysics literature on power laws in financial economics. Gabaix et al. [11] investigated how Power laws describe histograms of financial fluctuations, such as in stock price, trading volume and the number of trades; they observed that the exponents that characterize these power laws are similar for different types and sizes of markets; therefore, they proposed a model to provide explanation for these power laws. In another paper, Gabaix et al. [12] summarized some theoretical explanations for the two mechanisms leading to power laws including random growth and matching and economics of superstars. Bouchaud [13] also discussed on origins of power law distributions and correlations on financial time series, moreover Bouchaud explained the universality of the exponents describing price distribution and volatility correlation.

In a study, Gopikrishnana et al. [14] showed that the distributions of 5 min returns for 1000 individual stocks and the S&P 500 index decay as a power-law with an exponent around 3 outside the stable Levy regime between (0, 2). In another study, Plerou et al. [15] analyzed the fluctuations of the average bid–ask spread S using 116 most frequently traded stocks on the New York Stock Exchange over 1994–1995, and found S' decaying with power law distribution of exponent 3. Gopikrishnana et al. [16] studied the distribution of fluctuations of the S&P500 index return by analyzing three different databases including one containing 1min interval for 13 year period 1984–1996 data, and found out that the distribution behaves as a power law with an exponent of about 3. Bollerslev and Todorov [17] suggested a new nonparametric approach for estimating the jump tail; implemented on high frequency S&P500 index data, and obtained a value of 0.26 for the estimates for the jump-(left) tail parameters. Wu [18] also introduced a stylized model where jump arrival rate obeys an exponentially damped power law, and attempted to fit the model to S&P500 index return.

In this article, neither are we going to explore origins of power laws on financial time series [12,13], nor are we going to show that 5 min returns of stocks or indices decay a power law [14], but we will show that intraday jumps are power-law distributed ($\sim 1/r^{1+\mu}$) with the exponent $1 < \mu < 2$, in addition the time dynamics of returns after intraday jumps is power-law distributed, which indicates a long memory of return after jump. Consider that returns are composed of mixture of normal-distributed and power-law distributed data ($\sim 1/r^{1+\mu}$); the probability of rejection of null hypothesis for normality test is a function of μ, which is equal to one for $1 < \mu < 2$ within large intraday sample size. We prove this concept both analytically and empirically. In the analytical section, one utilizes the concepts of Realized Variance $RV = \sigma^2 + \sum J = \sum |r_j|^2 \approx M < |r|^2 >$ and Bipower Variation $BV = \sigma^2 = \sum |r_j||r_{j-1}| \approx M < |r| >^2$. In case $1 < \mu < 2$, RV goes to infinity while BV remains finite, therefore the jump can be detected. In the empirical section, we download S&P500 index data for both periods of 1997–1998 and 2014–2015, and show that the Complementary Cumulative Distribution Function (CCDF) of jump return(JR) is power-law distributed with the exponent $1 < \mu < 2$. In addition, it will be demonstrated that time dynamics of returns after intraday jumps decays as a power-law. We find out that time dynamics of returns after jump for financial data indicates a long memory behavior.

2. Theoretical development and propositions

2.1. Realized volatility and jump detection

Usage of realized volatility in financial economics goes back to many years ago, Poterba and Summers [19], Schwert [20] and many more. As already mentioned in the introduction, realized variance can be estimated as the sum of squares of financial returns. This theory presumes a stochastic volatility model for log-prices. A basic Brownian motion model can be generalized to allow time-dependent volatility. Log-price p follows the following dynamics:

$$dp(t) = \mu(t)dt + \sigma(t)\,dW(t) \tag{1}$$

where parameters σ and μ are to be stochastically independent of the standard Brownian motion W. Returns are defined by differencing the log-prices. Consider the time interval h (in our work, it is always a day, but it could be in general any