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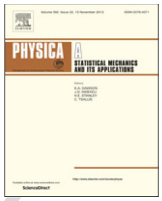

Optimal execution in high-frequency trading with Bayesian learning

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HIGHLIGHTS

- We propose a stochastic model to describe the optimal execution in high-frequency Trading.
- Traders' behaviors are described using Bayesian rules in the model.
- The explicit solutions to the stochastic model can be deduced by HJB equations.
- Our analysis gives the numerical solutions based on static and dynamic situation.

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ABSTRACT

We consider optimal trading strategies in which traders submit bid and ask quotes to maximize the expected quadratic utility of total terminal wealth in a limit order book. The trader's bid and ask quotes will be changed by the Poisson arrival of market orders. Meanwhile, the trader may update his estimate of other traders' target sizes and directions by Bayesian learning. The solution of optimal execution in the limit order book is a two-step procedure. First, we model an inactive trading with no limit order in the market. The dealer simply holds dollars and shares of stocks until terminal time. Second, he calibrates his bid and ask quotes to the limit order book. The optimal solutions are given by dynamic programming and in fact they are globally optimal. We also give numerical simulation to the value function and optimal quotes at the last part of the article.

1. Introduction

With the rapid development of electronic exchanges around the world, not only market-makers or specialists but any traders are willing to submit bid and ask quotes in limit order book as a dealer. In most markets, the availability of buying and selling on the same day makes it convenient for trader to do high-frequency trading. Under the circumstance of limit orders and high-frequency trading, many trading strategies rise in response to the conditions. Using the method of Bayesian learning and dynamic programming, this paper presents a new model for price dynamics and optimal execution.

The optimal execution has been studied widely in the literature. One way by which the dealer obtains an optimal strategy is to maximize his total wealth expected utility. Almgren and Chriss [1] study optimal execution using a quadratic utility function by adding Laplace coefficient into cost and risk equation. Another way is to minimize the expected cost of trading. Bertsimas and Lo [2] derive dynamic optimal strategies that minimize the expected cost of trading a large block of equity over a fixed time horizon. After that, Bertsimas et al. [3] present another paper, extending dynamic optimal trading strategies

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to the portfolio case in which price impact across stocks can have an important effect on the expected total cost of trading a portfolio. Isaenko [4] considers a portfolio optimization problem for a short-term investor under transitory price impact.

However, the hypotheses they consider are very simple. With the development of stock market and trading mechanism, many complicated issues are put forward. Iori et al. [5] introduce a microscopic model in order-driven markets. They propose that by limit orders, traders may be able to trade at a more favorable price. On the other side, limit orders can provide liquidity and improve the trading efficiency. They also states that if the market price is rising, the upward movements will trigger limit orders to sell; if the price is falling, the downward movements will trigger limit orders to buy. Zhou [6] studies the existence of two universal price-impact functions of two types of trades in an order-driven stock market, which does not depend on the stock capitalization. Ichiki and Nishinari [7] propose a simple stochastic order-book model for investors' swarm behaviors. Predoiu et al. [8] considers an optimal execution over a fixed time interval of purchasing a large asset in the face of a one-sided limit-order book. Besides, Bayraktar et al. [9] and Cebiroglu et al. [10] study the optimal order display and liquidation in limit order markets. Avellaneda and Stoikov [11] apply the exponential utility function to study high-frequency trading in limit order book, and get an approximate solution to the optimal trading strategies. However, due to the simple price dynamic model they give, their results turn out to have some unrealized shortages. In our framework, price dynamic follows Geometric Brownian Motion (GBM) in which the drift part is updated by Bayesian learning in the beginning of the transaction day. This modified model is intuitively better than Avellaneda and Stoikov [11] for the simple reason that we dynamically adjust our GBM model during the trading period, making our trade more advantaged.

Another set of aspects is the application of trader's trading strategies. Standard models of optimal execution use static strategy. An example of static strategy put forward by Huberman and Stanzl [12] is insider trading, in which dealers have already made an optimal strategy by illegal methods. One of the most famous static strategies is VWAP (Volume Weighted Average Price), which is calculated by adding up the dollars traded for every transaction (price times number of shares traded) and then dividing by the total shares traded for the day. Berkowitz et al. [13] consider VWAP to be a perfect way of obtaining and analyzing tradinginformation. Lert (2001) on the other way shows that VWAP benchmark underestimates trading cost brought by stock price trend. Indeed, the static strategy makes traders more passive in trading process, because they cannot calibrate trading strategy to the direction they want. In order to satisfy this requirement, Almgren and Lorenz [14–16] came up with a new trading strategy called Adaptive Trading. They separate a whole trading period into two parts and make a "rule" before trading. In the first part, dealers collect information and study the price movement. In the second part, strategy will be improved due to the useful information from the first part. In this paper, transaction period also is divided into two parts. People first observe statistical law of drift factor in price dynamic, making an estimation of drift parameter. Then, the trader could adjust his price movement by Bayesian theorem. Moallemi et al. [17] present an algorithm for computing perfect Bayesian equilibrium behavior and conduct numerical experiments.

The paper is organized as follows. In Section 2, we construct a theoretical stochastic model using Bayesian theorem to describe the issue of optimal execution and then the solutions to the optimal bid and ask quotes are given. Section 3 contains some numerical simulations to our solutions. At last, we present some conclusions of our work in Section 4.

2. Model

2.1. The stock price with Bayesian learning

A limit order contains bid and ask price, which are all evolved according to the continuous trading. The bid price $p_1(t)$ is the highest price that a bidder is willing to purchase the stock and the ask price $p_2(t)$ is the lowest price a seller wants to sell the stock. We assume the mid-price to be the market price of the stock. In the trading period, the trader only trades a single asset whose mid-price is $P(t)$,

$$P(t) = \frac{p_1(t) + p_2(t)}{2} \tag{1}$$

obeying an Geometric Brownian Motion (GBM):

$$\frac{\mathrm{d}P(t)}{P(t)} = \alpha \mathrm{d}t + \sigma \mathrm{d}B(t) \tag{2}$$

with $P(0) = P_0$. Here $B(t)$ is a standard one-dimensional Brownian motion, σ refers to the volatility and α a drift factor. This continuous-time model shows that we do not consider any autocorrelation structure for the stock. The volatility results from the ignorant retail investors who have no information about market. This kind of behavior could be traced by history information, so we assume σ to be a constant. The drift factor changes on account of institutional investors who have already made strategies before trading. When the institutional investors intend to buy stocks, the price will rise, and vice versa.

We could therefore previously predict institutional investors' preference due to the stock price movement. We assume that the strategy is static like VWAP, so α is a constant even though we do not know its real value. In the beginning of the trading period, we assume a prior normal distribution of drift factor

$$\alpha \sim N(\overline{\alpha}, v^2) \tag{3}$$

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