Physica A xx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Q1 Probabilistic analysis of cascade failure dynamics in complex network☆

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HIGHLIGHTS

- Nonidentical tolerance parameters are considered in the Load-Capacity model.
- A probabilistic cascade failure model is proposed to describe the evolution of blackout scale in time.
- The critical attack size ensuring no collapse is obtained.

ARTICLE INFO

Article history:
Received 6 February 2016
Received in revised form 4 May 2016
Available online xxxx

Keywords:
Mean field
Probabilistic cascade failure
Typical distributions
Damage
Complex network

ABSTRACT

The impact of initial load and tolerance parameter distribution on cascade failure is investigated. By using mean field theory, probabilistic cascade failure model is established. Based on the model, the damage caused by certain attack size can be predicted, and the critical attack size is derived by the condition of cascade failure end, which ensures no collapse. The tolerant critical attack size is larger than the case of constant tolerance parameter for network of random distribution. Comparing three typical distributions, simulation results indicate that the network whose initial load and tolerance parameter both follow Weibull distribution performs better than others.

1. Introduction

Complex network theory has attracted much attention due to its wide application in many areas such as multi-agent systems and power grids [1–6]. Due to the complexity of modern systems, some slight glitches often give rise to cascade failures. Without efficient manual intervention, cascade failures may lead to large-area blackouts such as the one that occurred in northeastern US and eastern Canada in 2003 [7], and the accident in India in 2012 [8], which have caused tremendous fear among people and should be avoided as far as possible. Considering the disaster damage and economic loss, much effort has been devoted to studying the propagation behavior of cascade failures and developing efficient strategies to protect networks [9–12]. Q3

☆ This work was supported in part by the National Natural Science Foundation of China under Grants 61170031, 61272069, 61373041, 61572208 and 61272114.

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http://dx.doi.org/10.1016/j.physa.2016.05.059
0378-4371/

Networks may suffer great damage from light initial attack due to cascade failure. The attack in a network can be random Q4 and target [13–15]. In Ref. [16], cascade failure model based on shortest path redistribution is constructed for scale-free networks, the results suggest that the attack on a node of high degree may trigger cascade failure, which is capable of disabling the network almost entirely and the network performs better if it has larger tolerance parameter. Moreover, the significance of the degree of a node is evaluated in Ref. [17], and an optimal capacity allocation is proposed based on node significance which improves the capacity of those significant nodes, and reveals that robustness of the network can be further enhanced by choosing proper capacity allocation parameter for different tolerance parameters. However, since real networks are great and growing such that the topologies are not completely known, the above method on shortest paths redistribution remains to be questioned. Therefore, neighbor node redistribution [18,19] arises. The tolerance of cascaded failures in weighted networks is investigated in Ref. [20], three weighting strategies including node degrees [21], node betweenness centrality [22] and edge betweenness centrality are considered. The results indicate that the networks with edge betweenness own the best robustness, and it suggests that the load distribution of links plays an important role in the robustness of the network against cascade failure. Besides, available historical blackout data [7,8], and the analysis of line overloads in Refs. [23–25], all testify the important status of transmission lines.

The ORNL-PSerc-Alaska (OPA) model is proposed in Ref. [26] to investigate the blackouts caused by line outages. Furthermore, Southeast European power transmission system is analyzed as an evolving grid in Ref. [27], where the OPA model is adopted. In Ref. [28], a continuous-time Markovian chain is introduced to model the cascade-failure dynamics, which retains key physical attributes and operate characteristics of power grids. Notably, in Ref. [28] the authors assume that each interval allows only one faulty line increased, which is not realistic. In fact, [29] suggests that local cascade models cannot exactly describe power grids or flow networks. On the other hand, global load redistribution is better in describing cascade failure and even more disruptive than percolation attacks, up to the point of first-order on a single network and even interdependent networks [30–34]. Besides, [35] concretizes the evolution of cascade failure in Ref. [31] under a certain proportion of random attack, meanwhile, it provides a detail analysis of three specific load distributions, by contrast, it finds that Weibull distribution is the best (in terms of critical attack size) while Pareto is the worst.

In fact, the capacity of a transmission line is affected by its initial load and tolerance parameter, where it will be a waste to assign a fixed tolerance parameter for each line [36] as well as adverse to the stability of networks [37]. Inspired by the above discussions, the affect of initial load and tolerance parameter distribution on cascade failure is investigated in this paper. We analytically construct a probabilistic cascade failure model to describe the evolution of cascade failure, and then derive the relationship between initial attack and final survived lines. The main contributions of this paper are as follows: (i) Compared with the works in Ref. [16] where the tolerance parameters are equal for each transmission line, we consider the tolerance parameters are different and follow certain distribution. This is proved to be of less cost and more resilient for networks in Refs. [36,37]. (ii) Different from the stochastic abstract-state evolution model in Ref. [28], we construct a probabilistic cascade failure model based on global load redistribution, which enables the evolution of cascade failure in real time under certain attack size, and the faulty lines at each interval are not restricted. (iii) The critical attack size ensuring no collapse is obtained by our probabilistic cascade failure model, which can be used as an indicator measuring the performance of networks. By introducing three typical distributions, our simulation results suggest that the network whose initial load and tolerance parameter both follow Weibull distribution performs better than others.

This paper is organized as follows. A novel probabilistic model of cascade failure is established in Section 2. In Section 3, we present a damage analysis of above cascade failure model and the critical attack size is derived. In Section 4, three typical distributions are introduced and some comparison results are shown. Finally, Section 5 reports some concluding remarks.

2. Probabilistic cascade failure model

In this section, we discuss the cascade failure problem with certain probability of initial failure lines f_0. A Probabilistic Cascade Failure Model based on mean field theory will be established, which describes the evolution of cascade failure and allows the prediction of damage in real time. The detailed analysis is in the following.

Consider a network consisting of N lines with interdependent initial loads $L_1, L_2, \ldots, L_N$ which follow a distribution $\mathbb{P}_L(x) = \mathbb{P}(L \le x)$, $\mathbb{E}(\cdot)$ indicates the corresponding mathematical expectation. Ω is the set of all the lines, and Ψ denotes the set of faulty lines at stage 0. $\mathbf{1}_A(x)$ is indicator function, i.e. $\mathbf{1}_A(x) = 1$ if $x \in A$, and $\mathbf{1}_A(x) = 0$, otherwise.

The capacity of a line is defined as the maximum flow which can be handled. Similar to Ref. [16], the capacity C_i of line i is assumed as

$$C_i = (1+\alpha_i) L_i, \quad i = 1, 2, \ldots, N, \tag{2.1}$$

where $\alpha_i > 0$ is the tolerance parameter of line i which cannot be infinite because of limited costs.

Remark 2.1. The capacity of a line is related to its initial load as well as tolerance parameter. In Ref. [35], transmission lines in network own varied initial load but fixed tolerance parameter. However, the tolerance parameter is related to economic cost and represents the importance of a line in the whole network. Fixed tolerance parameter for each line may lead to waste and be adverse to the stability of network. Therefore, it is more reasonable to assume that different lines have different tolerance parameters.