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Q1 Co-evolution of payoff strategy and interaction strategy in prisoner's dilemma game

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HIGHLIGHTS

- The payoff as a kind of evolving strategy in prisoner's dilemma game is considered.
- The co-evolution of interaction strategy and payoff strategy in prisoner's dilemma game is investigated.
- Single-peak or double-peak, even a fake three-peak structure may show up in final distribution of the payoff strategy.

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ABSTRACT

Co-evolutionary dynamical models, providing a realistic paradigm for investigating complex system, have been extensively studied. In this paper, the co-evolution of payoff strategy and interaction strategy is studied. Starting with an initial Gaussian distribution of payoff strategy r with the mean u and the variance q, we focus on the final distribution of the payoff strategy. We find that final distribution of the payoff strategy may display different structures depending on parameters. In the ranges $u < -1$ and $u > 3$, the distribution displays a single-peak structure which is symmetric about $r = u$. The distribution manifests itself as a double-peak structure in the range $-1 < u < 3$ although a fake three-peak structure shows up in range $1 < u < 2$. The explanations on the formation of different types of payoff strategy distributions are presented.

1. Introduction

Co-evolutionary game theory has been a mushrooming framework for addressing the dynamic evolution of nonlinear complex systems since the initial works by Zimmermann et al. [1]. It is a natural development of evolutionary games since in reality besides the strategies, environment and many other factors evolve in time as well and affect back the outcome of the evolution of strategies in turn [2]. In co-evolutionary game theory, the prisoner's dilemma game (PDG) [3] is one of the most widely employed. In a typical two-player, one-shot PDG, the players' payoffs depend on the *interaction strategy*, cooperation or defection, of both players. If both players choose to cooperate, they receive reward R, whereas if both decide to defect, they receive punishment P. The cooperator receives a suckers payoff S when her opponent exploits a temptation T by choosing defection.

Co-evolutionary PDGs involving the evolution of strategy and the links between players [4–10], the teaching ability of players [11–14], the motion of players [15–20], and other factors [21–23] have been extensively studied. Recently, the effects of payoff on strategy-decision are investigated [24,25]. In Ref. [24], players whose payoffs from the previous round exceed a threshold adopt only a minimally low temptation to defect in the next round and they found that the lower the threshold for

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using the small temptation to defect, the more the evolution of cooperation is promoted. In another work [25], a fraction of the population adopts either a positive or a negative value of the suckers payoff and the results showed that the higher the fraction of the population adopting a different payoff matrix the more the evolution of cooperation is promoted. However, payoff as a kind of strategy which could evolve and be learned in the game has not been wide studied yet. In this paper, we assume the payoff as a kind of evolving strategy and study the co-evolution of interaction strategy and payoff strategy in prisoner's dilemma game. We define that the payoff T equals $1+r$ and S equals $1-r$ as shown in payoff matrix, where r, named *payoff strategy*, is assigned of different values following Gaussian distribution for different players. We focus on the effects of the co-evolution on cooperation behaviors and the final payoff strategy distribution. It turns out that the original Gaussian distribution of payoff strategy r evolves eventually into a double-delta distribution which is roughly symmetric in part of given parameter regimes. Whereas, in other regimes, the final distribution of payoff strategy is still a single-peak function similar to the initial Gaussian function. We make detailed analysis and explanation which are in agreement with those results we gain.

The paper is structured as follows. Section 2 is devoted to the description of the co-evolutionary spatial prisoner's dilemma game. In Section 3, phenomena and mechanism for the final distribution of r are presented, while in Section 4 we summarize the results.

2. Model

We consider an evolutionary game on a square lattice with periodic boundary conditions. The size of the lattice is set to be $N = 100 \times 100$ and each node in the lattice is occupied by a player which takes one of the interaction strategies, cooperation (C) or defection (D). Initially, players are assigned with cooperation or defection with equal probabilities. Each player will play games with his nearest neighbors by following the payoff matrix

A\B	C	D
C	1	1-r
D	1+r	0

.

Different from the ordinary way where the game parameter r is the same for all players, here r, payoff strategy, is a property of players, which is initially drawn from the Gaussian distribution

$$f(r) = \frac{1}{\sqrt{2\pi}q}\exp\left(-\frac{(r-u)^2}{2q^2}\right) \tag{1}$$

with u the mean and q the variance.

In each Monte Carlo (MC) simulation time, a randomly selected player A interacts with his nearest neighbors to earn his payoff P_A. Meanwhile, four nearest neighbors of player A accumulate their payoffs in the same way. Then player A updates his interaction strategy with the probability v or his payoff strategy with the probability $1-v$ by following one of his neighbor B. The transition rate in the strategy updating takes the form of the Fermi rule

$$W = \frac{1}{1+\exp[(P_A - P_B)/k]}$$

where the parameter k characterizes the uncertainty in the decision-making of players and is set to be $K = 0.1$.

3. Result

In this work, we concern with the final distribution of the payoff strategy r when the steady state of the interaction strategies in population has been reached and the effects of parameters u and q on it. Through the work, we let $v = 0.5$ and the extensive simulations reveal that v does not change the results qualitatively provided that $v \neq 0$ and $v \neq 1$.

A typical final distribution of r, $P(r)$, is presented in Fig. 1 where $u = 0$ and $q = 1$. Clearly, the initial Gaussian distribution evolves into a one with the double-delta function. In other words, only two different payoff strategies are left after the evolution in each simulation. To be mentioned, the ensemble average of $P(r)$ over different realizations is symmetric about $r = 0$ at $u = 0$ and $q = 1$, which will be shown in Fig. 3(a).

To get a microscopic view on how the distribution of r evolves, we present the snapshots of the spatial pattern of r at different time steps in Fig. 2 for the same parameters as Fig. 1. In the beginning, players with small $|r|$ marked by light green and light blue are dominant and those with high $|r|$ marked by red and dark blue are sparsely scattered over the lattice. When time goes on, players with high $|r|$ increase rapidly by changing payoff strategies of their neighbors, which reflects the fact that players with higher $|r|$ always have higher payoff than their neighbors. As shown in the snapshot at $t = 50$, players with medium $|r|$ marked by gray blue and yellow form tiny clusters. The clusters grow up in size with time but with complicated boundaries (see the snapshots at $t = 200$). Finally, those clusters break up when only two largest $|r|$ are left and the players with two kinds of r are intermingled with each other over the lattice.

Furthermore, we study the effects of the mean u and the variance q on the final distribution of r. The ensemble average of the final distribution $\langle P(r)\rangle$ is obtained for 100 realizations. In Fig. 3(a), $\langle P(r)\rangle$ against u is presented for $q = 1$ and, to be

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