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Multiple commodities in statistical microeconomics: Model and market

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HIGHLIGHTS

- The Empirical study of statistical pricing model is investigated.
- The Pricing of multiple commodities is made by the empirical model.
- The Pricing of single commodities is described by the empirical model.
- The Cross-Correlation of different commodities is determined by the model.
- The Auto-correlation of single commodities is studied by the model.

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ABSTRACT

A statistical generalization of microeconomics has been made in Baaquie (2013). In Baaquie et al. (2015), the market behavior of single commodities was analyzed and it was shown that market data provides strong support for the statistical microeconomic description of commodity prices. The case of multiple commodities is studied and a parsimonious generalization of the single commodity model is made for the multiple commodities case. Market data shows that the generalization can accurately model the simultaneous correlation functions of up to four commodities. To accurately model five or more commodities, further terms have to be included in the model. This study shows that the statistical microeconomics approach is a comprehensive and complete formulation of microeconomics, and which is independent to the mainstream formulation of microeconomics.

1. Introduction

The theory of prices proposed in Ref. [1] is based on the concept of the action functional; the subsequent publication [2] provides strong empirical evidence in support of this formulation for the case of single commodities. The present paper extends the analysis to multi-commodities by modifying the single commodity model in a parsimonious manner.

The theory of commodity prices [3] is one of the bedrocks of microeconomics and usually starts with the concept of the utility function of a typical consumer [4,5]. A maximization of the utility function with a budget constraint yields the demand for the commodities as a function of price. The supply function is obtained by maximizing the profit for the producers and the market prices of commodities in conventional microeconomics are fixed by equating supply with the demand [4,5].

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In contrast to conventional microeconomics, in statistical microeconomics [1] the prices of all commodities are taken to be intrinsically random—and the probability distribution function of prices is fixed by the exponential of the so called *action functional*. The action functional in turn is the sum of two parts, a 'kinetic' term that determines the dynamical evolution of commodity prices and a microeconomic potential that is the sum of the supply and demand functions. The action functional contains all the information of the market and determines the distribution of market prices as well as the change in market prices as the prices evolve in time [6–8].

The primary focus in the statistical microeconomic formulation is to describe the unequal time correlation functions of market prices. The auto- and cross-correlation [9,10] functions for multiple commodities is modeled using the action functional and the Feynman path integral. The action functional is calibrated by matching the prediction of the model's correlation functions with the observed market and provides a stringent test of the accuracy of the model.

The microeconomic potential for commodity with price p is given by $\mathcal{V}[p]$ and has been introduced in Ref. [1]; the potential has its minimum value at its extrema $\hat{p}$, given by

$$\partial\mathcal{V}[\hat{p}]/\partial p = 0.$$

The price $\hat{p}$ is taken to be the average commodity price.

What happens when the price p is not equal to the average price $\hat{p}$, that is, $p \neq \hat{p}$? The microeconomic potential $\mathcal{V}[p]$ in this case causes the prices to 'move', that is, to change and tend towards $\hat{p}$. Clearly, the more abrupt the change, the more unlikely it is; the change of price should, for normal market conditions, be gradual and relatively 'smooth'. To achieve this smooth movement of the prices in general, a 'kinetic term' $\mathcal{T}[\mathbf{p}(t)]$ is introduced. Although the concept of the kinetic term is taken from physics, it finds a natural expression in the evolution of the prices of commodities: the specific form of the kinetic term is determined by the study of market data.

The kinetic term in the action functional is seen to be strongly supported by market data, and as of now has no clear theoretical explanation. One can only speculate that demand and supply are determined by consumers and producers, respectively and that the kinetic term reflects the process of circulation, distribution and exchange – as well as the degree of market liquidity – that is necessary for the products to make a transition from the producer to the final consumer in the market.

One rather unexpected result is that the kinetic term in the action functional has a dominant role in the evolution of commodity prices; due to the high time derivative of prices in the kinetic term, the short term evolution of commodity prices is completely dominated by the kinetic term, with the microeconomic potential, containing the supply and demand functions, that come into play for the long term evolution.

2. The microeconomic action functional

Consider N commodities, with market prices given by $p_I;\ I = 1, \ldots, N$. Prices are always positive and can be represented by exponential variables as $p_I = p_0 e^{x_I}$; the normalized logarithm of prices, denoted by x_I, is defined as follows

$$p_I = p_{0I} e^{x_I}; \qquad x_I(t) = \ln(p_I(t)/p_{0I}); \qquad I = 1, \ldots, N.$$

The demand function and the supply function are modeled to be [1]

$$\mathcal{D}[\mathbf{p}] = \sum_{i=1}^{N} \tilde{d}_i p_{0i} e^{-\tilde{a}_i x_i}; \qquad \mathcal{S}[\mathbf{p}] = \sum_{i=1}^{N} \tilde{s}_i p_{0i} e^{\tilde{b}_i x_i}; \quad \tilde{d}_i, \tilde{s}_i > 0;\ \tilde{a}, \tilde{b} > 0. \tag{1}$$

The coefficients $\tilde{d}_i, \tilde{s}_i$, according to Ref. [1], are determined by macroeconomic factors such as interest rates, unemployment, inflation and so on.

For the purpose of modeling, prices in statistical microeconomics are expressed in terms of variables that are measured from the average value and normalized by the volatility of the stock.

$$y_i(t) = \frac{x_i(t) - \bar{x}_i}{\sigma_i}; \quad i = 1, \ldots, N \tag{2}$$

$\bar{x}_I$ and σ_I are the average value of y_I. The volatility of $x_I(t)$ for the time period being considered and are given by

$$\bar{x}_i = E[x_i]; \qquad \sigma_i^2 = E[\left(x_i - \bar{x}_i\right)^2].$$

The normalized variables y_i are all of $O(1)$ and hence one can model and compare commodities with vastly different volatilities and prices. In the statistical microeconomic approach, the microeconomic potential is the fundamental quantity that combines supply and demand by considering their sum [1]. The supply and demand yield the microeconomic potential given by

$$\begin{aligned}\mathcal{V} &= \sum_{i=1}^{N} \left[\tilde{d}_i p_{0i} e^{\tilde{a}_i \bar{x}_i} e^{-\tilde{a}_i \sigma_i y_i} + \tilde{s}_i p_{0i} e^{-\tilde{b}_i \bar{x}_i} e^{\tilde{b}_i \sigma_i y_i}\right] \\ &\equiv \sum_{i=1}^{N} \left[d_i e^{-a_i y_i} + s_i e^{b_i y_i}\right] \end{aligned} \tag{3}$$