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Biased random walk in spatially embedded networks with total cost constraint

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HIGHLIGHTS

- This paper studies random walk with a bias in spatial network with total cost restriction.
- This paper finds that the best optimal transport is obtained with an exponent $\alpha = d + 1$ for all p.
- The special phenomena can be possibly explained by the theory of information entropy.

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ABSTRACT

We investigate random walk with a bias toward a target node in spatially embedded networks with total cost restriction introduced by Li et al. (2010). Precisely, The network is built from a two-dimension regular lattice to be improved by adding long-range shortcuts with probability $P(r_{ij}) \sim r_{ij}^{-\alpha}$, where r_{ij} is the Manhattan distance between sites i and j, and α is a variable exponent, the total length of the long-range connections is restricted. Bias is represented as a probability p of the packet or particle to travel at every hop toward the node which has the smallest Manhattan distance to the target node. By studying the mean first passage time (MFPT) for different exponent log $\langle l \rangle$, we find that the best transportation condition is obtained with an exponent $\alpha = d + 1 (d = 2)$ for all p. The special phenomena can be possibly explained by the theory of information entropy, we find that when $\alpha = d + 1$ $(d = 2)$, the spatial network with total cost restriction becomes an optimal network which has a maximum information entropy. In addition, the scaling of the MFPT with the size of the network is also investigated, and finds that the scaling of the MFPT with L follows a linear distribution for all $p > 0$.

1. Introduction

Many real complex networks can be geographically represented or spatially embedded [1], such as social networks [2,3], the global airport network [4,5], wireless communication networks [6], physical systems [7] and also the network of activity in the brain [8,9], and so on. Both theoretical and empirical studies have revealed that spatially embedded networks structure and dynamics on networks exhibit interdependent relationships with each other, which is actually an important and fundamental problem in the field of complex networks [1,10,11].

In recent studies, it has been shown that the navigability of the spatially embedded networks can be influenced by the geometrical structure of networks [12–19]. Generally speaking, navigation in complex networks with global information is the most efficient way to transport a particle from the source to the target. But in reality, it is hard to get the global

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information from the networks. Because of this, a practical scheme is proposed, and has caught many scientists' attention, which is navigation with local information. First, Kleinberg established a model to study optimal navigation with local knowledge in a small-world network by using greedy algorithm [12]. Kleinberg considered an $L \times L$ square lattice, where each node is connected with its neighbors and randomly generates a long-range connection with a probability $P(r_{ij}) \sim r_{ij}^{-\alpha}$, where r_{ij} is the Manhattan distance between sites i and j, and α is a variable exponent, and found that the small-world feature of the network can only be efficiently accessed if the exponent $\alpha = d$ [12]. Later, Roberson et al. studied the navigation problem in fractal small-world networks [13], where they proved that $\alpha = d$ is also the optimal power-law exponent in the fractal case. And then Cartozo et al. used dynamical equations to study the process of Kleinberg navigation [14,15]. They provided an exact solution for the asymptotic behavior of such a greedy algorithm as a function of the dimension d of the lattice and the power-law exponent α. Thadakamalla et al. proposed several decentralized search algorithms, including greedy algorithm, on the spatial scale-free networks, and found that the spatial scale-free network is searchable for a wide range of parameter space [16]. Moreover, M. Boguñá et al. showed that specific structure features of many complex networks support efficient transportation without global information, such as greedy algorithm, and indicated that real networks may have hidden metric space that undiscovered [17].

Recently, based on Kleinberg's spatial networks, Li et al. proposed a cost constraint on the total length of the additional links [18,19]. They found an interesting phenomena that the best transport condition is obtained with a power-law exponent $\alpha = d + 1$ for both local and global navigation. Here comes the question, is the optimal transport condition also $\alpha = d + 1$ if some nodes or all nodes of the networks hold null information about the structure of the network? In order to answer this question, we consider a biased random walk navigation strategy, the bias is represented as a probability p of the packet or particle to travel at every hop toward the node which has the smallest Manhattan distance to the target node. That is, navigating with a probability p to follow the greedy algorithm and $1 - p$ to follow random walk algorithm. This condition can help us to optimize the navigation in a spacial network such as social network and airline network. By investigating the MFPT for different exponent α, we find that the best transportation condition is obtained with an exponent $\alpha = d+1$ for all p. The special phenomena can be possibly explained by the theory of information entropy, we find that the spatial network with total cost restriction is an optimal network with a maximum information entropy when $\alpha = d + 1$.

2. Model

2.1. Generating the network

Based on Kleinberg's spatial networks, Li et al. propose a spatial network model with total cost restriction [18,19]. Fig. 1 shows a typical spatial network with total cost restriction, which is a regular two-dimensional square lattice with $N = L \times L$ node, where L is the linear size of the lattice, each node is connected with its four nearest neighbors. In the model, pairs of nodes i and j are randomly chosen to receive long-range connections with probability $P(r_{ij}) \sim r_{ij}^{-\alpha}$, where r_{ij} is the Manhattan distance between nodes i and j. And the total length of the long-range connections is restrict by $\Lambda = CL \times L$. The probability $P(r_{ij})$ that nodes i and j will have a long range connection can be mapped on a density distribution $p(r)$, where $r = r_{ij}$. The number of nodes separated by a lattice distance r from a given node in a d-dimensional lattice is proportional to r^{d-1}. Thus the distance distribution of the long-range connections $p(r) \sim r^{-\alpha} r^{d-1}$ can be normalized as $\int_1^L p(r)dr = 1$, which yields

$$p(r) = \begin{cases} (d-\alpha)\dfrac{1}{L^{d-\alpha}-1} r^{-\alpha} r^{d-1}, & \alpha \neq d, \\ \dfrac{1}{\ln L} r^{-\alpha} r^{d-1}, & \alpha = d. \end{cases} \tag{1}$$

And then, the distance r can be obtained from random numbers $o < u \leq 1$ chosen from the uniform distribution, by

$$r = \begin{cases} [1 - u(1 - L^{d-\alpha})]^{1/(d-\alpha)}, & \alpha \neq d, \\ L^u, & \alpha = d. \end{cases} \tag{2}$$

The network model can be generated following algorithm in Refs. [19,20]:

(i) Creating a regular d-dimensional lattice with N nodes with each node connected to its $2d$ nearest neighbors. (in this paper, $d = 2$).
(ii) We randomly chose a node i to create a long-range connection. The length of the long-range connection r $(1 < r \leq L)$ is randomly selected using Eqs. (2). We consider all N_r nodes on the Manhattan distance $S = [r]$ (if $r - [r] > 0.5$, then $S = [r] + 1$, and if $r - [r] \leq 0.5$, then $S = [r]$) from node i, that are not yet connected to node i.
(iii) We randomly pick one node j from the N_r nodes, and then connect nodes i and j.
(iv) Return to step (ii), until the total length of the long-connections reaches the preset cost Λ, e.g. $\Lambda = L \times L$. This algorithm ensures that the desired distribution function $p(r) \sim r^{-\alpha} r^{d-1}$ is fulfilled.

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