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Pinning lag synchronization of drive–response complex networks via intermittent control with two different switched periods

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HIGHLIGHTS

- The pinning lag synchronization of two complex networks is studied.
- An intermittent strategy with two different switched periods is developed.
- Some low-dimensional conditions ensuring lag synchronization are proposed.
- The minimum number of pinned nodes can be calculated.

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ABSTRACT

This paper develops an intermittent control with two switched periods to study the pinning lag synchronization problem of drive–response complex networks. These two switched periods may have different control rates. By designing appropriate adaptive intermittent pinning controllers and using Lyapunov stability theory, some sufficient conditions for ensuring the lag synchronization between two coupled networks are derived. The minimum number of pinned nodes is determined by node dynamics, coupling strength, inner coupling matrix and a design parameter. Two simple pinning lag synchronization criteria are obtained from the proposed conditions. A numerical example is provided to illustrate the feasibility of the theoretical results.

1. Introduction

In the past few decades, complex networks have received considerable attention by many researchers owing to the wide applications in science, engineering and society systems [1–3]. In fact, complex network models usually exist in many real-world systems, such as Internet, ecological network, metabolic network, biological networks, traffic network, and social network. Among many dynamical behaviors occurring in complex networks, synchronization is an important and interesting phenomenon. On the one hand, as a typical collective behavior in complex networks, synchronization can be used to demonstrate many natural phenomena, such as the lighting of fireflies and the spread of an epidemic. On the other hand, synchronization of complex networks has various potential applications, including secure communication, image processing, etc.

Up to date, synchronization in large-scale networks consisting of coupled dynamical nodes has been widely and deeply investigated [4–14]. Since a dynamical network usually has of a large set of interconnected nodes, it is difficult and

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impossible to add control actions to all nodes. In order to reduce the number of controlled nodes, a direct idea is to select a small fraction of nodes and add controllers to these selected nodes to achieve synchronization. This can be classified as pinning control strategy. Due to its advantage in reducing corresponding cost, pinning control technique has been broadly applied in the study of network synchronization, and many meaningful achievements have been acquired [15–20].

Recently, intermittent control, as a discontinuous control strategy in engineering fields, has been employed to realize synchronization of complex dynamical networks. In this type of control strategy, each period usually contains two types of time, one is work time where the controller is activated, and the other one is rest time where the controller is off. When rest time is zero, the intermittent control reduces to continuous control, while work time is zero means that the intermittent control becomes the impulsive control. To save the control cost and amount of transmitted information effectively, combining pinning control and periodically intermittent control has been extensively investigated for achieving network synchronization by researchers in recent years. In the traditional periodically intermittent control strategy [21–24], the control period, the control time as well as the control rate was assumed to be constants. This requirement of periodicity in intermittent control is unreasonable, which can unavoidably lead to some conservatism in practical applications. In Ref. [25], the authors discussed the pinning synchronization problem in complex networks with both non-delay and delay couplings by using intermittent control with two switched periods. Compared to intermittent control with one period, the proposed control method is less conservative and more practical. Furthermore, a new semi-periodically synchronization control scheme for switched complex networks is proposed in Ref. [26], in which intermittent control with multiple control periods was used to handle the synchronization problem in complex networks. Obviously, this new method is more flexible, and its application scope is wider. Inspired the non-periodical intermittent control strategy, Liu et al. [27,28] discussed the synchronization problem of complex dynamical networks via aperiodically intermittent pinning control.

Note that the aforementioned synchronization behaviors occurring in a single network, i.e., the synchronization of all nodes within a network, which was termed as inner synchronization in Ref. [29]. With the deepening and development of theories and applications of complex networks, many researchers are interested in the study of synchronization between two coupled complex dynamical networks, which can be regarded as outer synchronization [29]. In nature, much phenomenon can be used to explain outer synchronization between two networks, such as AIDS, mad cow disease, bird flu, SARS and so forth. In the past few years, many different kinds of outer synchronization behavior were proposed and a series of interesting and important results have been reported in this field [30–38].

As is well-known, because of the finite speed of information transmitting, a propagation delay inevitably needs to be considered in the implementation of dynamical systems. Lag synchronization, as a meaningful outer synchronization phenomenon, implies that the state of one work is delayed by a limited time to another network. In Ref. [39], the pinning lag synchronization problem was first considered and some sufficient conditions were derived. By using impulsive control, several criteria were proposed to ensure the lag synchronization between two general dynamical networks in Ref. [40]. Based on linear feedback and adaptive feedback control, Sun et al. in Ref. [41] investigated the pinning lag synchronization between two networks. In Ref. [42], the projective lag synchronization between two delayed networks was investigated under hybrid feedback control.

It is worth mentioning that the previous works mainly focused on lag synchronization between two coupled networks via pinning control or impulsive control [39–41]. There are few results concerning with lag synchronization problems under intermittent control [43–45]. Furthermore, taking the control cost and practical implementation into account, it is of great importance to study the lag synchronization between two complex networks by combining intermittent control and pinning control. However, to the best of our knowledge, there have not been any analytical results about the pinning lag synchronization in drive–response complex dynamical networks by using intermittent control, especially intermittent control with two switched periods.

Motivated by the above discussions, this paper devotes to investigate the pinning lag synchronization problem between two coupled complex networks by using intermittent control. The intermittent strategy includes two switched periods with different rates of control duration. By adding adaptive semi-periodically intermittent controllers to a small fraction of nodes in response network, several sufficient conditions are derived based on the Lyapunov stability theory and strict mathematical proofs. As a result, the lag synchronization between two complex networks can be obtained. Moreover, we can find that the minimum number of pinned nodes is determined by node dynamics, coupling strength, inner coupling matrix and a designed parameter. Two simple intermittent pinning lag synchronization criteria are gained from the above conditions. A numerical example is given to demonstrate the effectiveness of the proposed scheme.

The rest of this paper is organized as follows. In Section 2, the model of drive–response complex dynamical network is presented and some preliminaries are also provided. Some criteria for ensuring lag synchronization between two coupled dynamical networks by adaptive semi-periodically intermittent pinning with two different switched periods are derived in Section 3. Numerical simulation is given in Section 4. Finally, a conclusion is drawn in Section 5.

2. Network model and preliminaries

Consider a complex dynamical network consisting of N identical nodes, which is characterized by

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} g_{ij}\Gamma x_j(t), \quad i = 1, 2, \ldots, N, \tag{1}$$