Physica A 462 (2016) 603–618

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A 3-states magnetic model of binary decisions in sociophysics

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HIGHLIGHTS

- A 3-states Blume–Capel magnetic model is proposed to understand social processes.
- Three topologically different networks of Small World type are studied in detail.
- Agent-based computer simulations confirm the corresponding analytical results.

ARTICLE INFO

Article history:
Received 26 October 2015
Received in revised form 31 March 2016
Available online 22 June 2016

Keywords:
Sociophysics
Three-states models
Small-World
Coalitions
Cooperation
Decision

ABSTRACT

We study a diluted Blume–Capel model of 3-states sites as an attempt to understand how some social processes as cooperation or organization happen. For this aim, we study the effect of the complex network topology on the equilibrium properties of the model, by focusing on three different substrates: random graph, Watts–Strogatz and Newman substrates. Our computer simulations are in good agreement with the corresponding analytical results.

1. Introduction

There is great interest in applying physical models of proven efficiency in the new field of sociophysics. Problems such as decision of living in a neighborhood [1], to go to a crowed bar [2] or to participate in a strike [3] are some particular examples of such applications.

Physics ideas in social science have been introduced in order to study hierarchical structures by using the principle of least difficulty [4], the effect of frustration for modeling the dissemination of culture within the Axelrod model in his energy landscape theory [5], and by Galam in his models of coalitions [6]. To study the emergence of order in such a system, magnetic models of the Ising family have been used [7–9]. In particular, within the social sciences, models have been developed to describe urban segregation, language change [10], business confidence and economic opinions [11]. These studies apply either the traditional two-states Ising model to binary choice or a Potts multistates model [12] to multiple choice. Representing opinions or positioning by discrete values, society can be modeled as a system in which an analogue of the magnetic spin variable represents the state of individuals while the couplings represent their interactions [13,14].

The Ising model is usually applied in the context of binary decisions, which can be to vote or not to vote, to buy or not to buy a certain good [15,16], etc. The Random Field Ising Model (RFIM) was first applied to social systems for studying

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http://dx.doi.org/10.1016/j.physa.2016.06.017

collective phenomena such as consensus and attitude changes in groups [17] and rational group decision making [18]. Another useful application of the RFIM in social and economics sciences is presented in Ref. [19], by taking into consideration heterogeneities and interaction in decision making, thus providing a unified framework to account for many collective socioeconomic phenomena leading to ruptures or crisis. Although in the mean-field approximation, quantitative details of the model might be sensitive to the type of the topology or the distribution of the idiosyncratic fields, the qualitative behavior does not depend very much on them [19].

By using coupled Ising models, the interdependent binary choices under social influence for homogeneous unbiased populations have been analyzed [20]. Currently, a comparison of the model with real data of the European labor market is under investigation [21].

Important results have also been obtained in Ref. [22], where real data concerning the drop of birth rates in European countries in the second half of the XXth century, the increase of cell phones in Europe in the 1990s and the way clapping dies out at the end of a concert have been successfully explained by using the RFIM.

Concerning three-states models, several real examples can be given, such as the decision to vote, not to vote or to abstain, to belong to NATO, to the Warsaw Alliance or to a third part Alliance, and so on. For example, Ref. [23] studies the opinion dynamics in a three-choice system dividing the population in random groups of fixed size, demonstrating that such a system always reaches an equilibrium, and in Ref. [24] there is a discussion on the opinion formation in a voter-like model of a 3-states agents system (yes, no, undecided), defined on a regular lattice.

The review [25] provides many examples of applications of statistical physics methodology and concepts for the study of social systems, including opinion formation, cooperation, cultural dynamics, language evolution, crowd behavior or human dynamics among many others.

On the other hand, modeling of Small World and scale-free networks allowed for powerful theoretical analyses of dynamical collective social phenomena, such as opinion formation, infection, or damage propagation in social networks, where the analysis of social structures and interactions is crucial. Both models can be combined to study dynamical processes on complex networks [8,26,27].

Activities like cooperation and coalition forming have been traditionally approached by game theory in politics and diplomacy [28], yet it is only recently that the concepts of Statistical Physics have been applied. In this context, various investigations have been done by representations in terms of cellular automata, [29,30] or in terms of magnetic models on a Cayley tree, similar to what we propose in this work [31,32]. In this article, we demonstrate that starting from a rather weak assumption, stating that each individual in the ensemble is an agent who can adopt three different states, we show that it is possible to effectively model a broad range of sociological processes. We model the society as a set of agents whose interactions attempt to minimize their frustration. In order to introduce the neutrality as a possible decision option, different to the symmetric choice, we use the Blume–Capel model [33,34]. Further, we make our model reside on an underlying complex network, which plays the role of substrate, and study the effects of its topology on the model behavior.

The Blume–Capel model has been applied in the context of urban segregation for the case of a fixed number of agents on a lattice [35] and for the case of an external reservoir of agents, thus modeling an open city [36]. Although the interpretation of the variables is different, in the sense that in the problem of opinion formation studied here the zero state corresponds to neutrality, while in the above cited references they correspond to an empty location, there are similarities among these models that will be commented later.

In this paper we will restrict ourselves to the mean-field case, in the sense that the agents perceive the opinion of the rest of the agents through the average opinion, thus the total "demand" becomes a public information that influences the individual agent.

The rest of the paper is organized as follows: in Section 2 we study magnetic models in which three states are allowed, in contrast with traditional Ising models; in Section 3 we study the network on which our model will be residing, with emphasis on the Annealed Network Approximation approach, which enables us to derive expressions for the order parameters on the grid substrate; Section 4 is devoted to the study of the order parameters and critical temperature of our three-states magnetic model on three different network substrates, with a special focus on the Newman substrate, which is deemed the most realistic. Finally, in Section 5 we discuss the results.

2. Three-states magnetic models

Blume, Emery and Griffiths (BEG) proposed the first version of their three-states model in Ref. [33] to explain the behavior of a ${}^3He - {}^4He$ mixture, which has been since successfully applied to a number of different problems.

The three states BEG model has been applied in the context of neural networks [37–39] giving way to enhanced techniques for information storage. It has been studied by a number of techniques: Bethe–Peierls approximation [40,41], real-space normalization [42] and exact recursion [43–46], among many others. In this paper we will use the mean-field approximation.

In analogy with the original model, we will consider an ensemble made up by agents of two species, neutral with spin 0 and non neutral with spin ±1, defining thus the following two parameters:

$$\mathcal{M} = \frac{1}{N} \sum_{i=1}^{N} \langle S_i \rangle \tag{1}$$