Physica A 461 (2016) 586–594

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Pattern selection and self-organization induced by random boundary initial values in a neuronal network

Jun Ma [a,*], Ying Xu [a], Chunni Wang [a], Wuyin Jin [b]

[a] Department of Physics, Lanzhou University of Technology, Lanzhou 730050, China

[b] College of Mechano-Electronic Engineering, University of Technology, Lanzhou 730050, China

HIGHLIGHTS

- Initial value-induced spatial regularity is observed.
- Boundary effect of network is discerned to detect the emergence of spirals.
- Coherence-resonance like behavior is found during pattern formation.

ARTICLE INFO

Article history:
Received 17 March 2016
Received in revised form 14 May 2016
Available online 14 June 2016

Keywords:
Neuronal network
Pattern
Spiral wave
Random condition

ABSTRACT

Regular spatial patterns could be observed in spatiotemporal systems far from equilibrium states. Artificial networks with different topologies are often designed to reproduce the collective behaviors of nodes (or neurons) which the local kinetics of node is described by kinds of oscillator models. It is believed that the self-organization of network much depends on the bifurcation parameters and topology connection type. Indeed, the boundary effect is every important on the pattern formation of network. In this paper, a regular network of Hindmarsh–Rose neurons is designed in a two-dimensional square array with nearest-neighbor connection type. The neurons on the boundary are excited with random stimulus. It is found that spiral waves, even a pair of spiral waves could be developed in the network under appropriate coupling intensity. Otherwise, the spatial distribution of network shows irregular states. A statistical variable is defined to detect the collective behavior by using mean field theory. It is confirmed that regular pattern could be developed when the synchronization degree is low. The potential mechanism could be that random perturbation on the boundary could induce coherence resonance-like behavior thus spiral wave could be developed in the network.

1. Introduction

The most prodigious characteristics of chaotic, hyperchaotic system could be that its dynamical behavior is much dependent on the selection of initial values. For most of the chaotic systems, the attractors can be reproduced completely when initial values are selected in the basin of attraction, no matter whether the system is dissipative or not. For example, it could encounter some difficulty in riddled basin of attractions. The potential mechanism could be that one positive Lyapunov exponent at least can be found in these chaotic systems with ergodicity of orbits. In fact, the rigorous proof of the ergodicity is a challenging task and hardly can be done in a general case (even for the well known Lorenz system). Furthermore, the

* Corresponding author.
E-mail address: hyperchaos@163.com (J. Ma).

http://dx.doi.org/10.1016/j.physa.2016.06.075

positive Lyapunov exponent may not imply chaos without ergodicity due to the Perron effects of LLE sign reversal [1]. Indeed, the selection of initial values can dominate the switch between chaotic and periodical states effectively. For example, Pham et al. [2] investigated the dynamics of memristive neural network with hidden attractors and realization in circuit was also practiced. The nonlinear circuit composed of memristor [3,2,4–6] can be controlled to reach chaotic state or periodical state by applying appropriate initial values, and these results can be verified by using bifurcation analysis. For example, Wu et al. [4] designed a nonlinear circuit composed of memristor, and used to generate periodical and bursting signals which could be of interesting for neuronal circuit. Hegab et al. [6] proposed a neuron model with memristive ionic channels being considered. For the nonlinear dynamical system and circuit coupled with memristor device, the developed state depends on the initial selection greatly. The mechanism is that one parameter such as conductance of memristor is dependent on the charge or magnetic flux, which is a variable of the dynamical system, so the parameter is variable-dependent. In our opinion, the dependence of initial selection could indicate the memory effect of device and even neuron [7].

In the case of network of neurons and array of coupled oscillators with certain topology connection, spatial distribution for observable variables in the network could give some insights to understand the collective behaviors of the network. A variety of spatial patterns such as spiral wave, target wave could be developed due to self-organization in the network, and these ordered waves can regulate the collective behaviors of network like continuous pacemaker. The network composed of identical oscillators or nodes can reach complete synchronization under appropriate coupling interaction and then the network presents homogeneous and synchronous states. However, regular distribution and spatial pattern such as target wave, spiral wave could be induced in the networks by applying external force, heterogeneity, or specific initial values in local area. For example, wedge-shaped initial values [8,9] are often used to generate spiral seeds thus spiral wave could be developed in excitable media. Furthermore, spiral wave is developed in reaction–diffusion system [8] by setting specific initial values and this scheme is also effective to develop spiral seeds and waves in neuronal networks [9]. As we known, various spatial patterns could be developed and transition of patterns occurs by changing certain bifurcation parameter or applying appropriate control schemes on the media [10–12]. For example, collective and synchronization behavior of network could be controlled by driving of autapses [13,14] connected to neurons. Within the previous studies, no-flux or periodical boundary condition [15] is often used for these networks with regular connection or small-world type of connection [16–24], and nodes on the boundary are often selected the same initial values. In fact, the diversity in initial values for nodes on the boundary could change the collective behavior of network greatly, which will be verified in this paper.

In this paper, a regular neuronal network is designed in a two-dimensional square array, which the local kinetics of node is described by the mathematical Hindmarsh–Rose [25] (HR) neuron model. The initial values for nodes on the boundary of the network are selected with random values while the other nodes are selected with the same value for quiescent state. The developed spatial distribution of membrane potentials of neurons in the network will be detected by observing the spatial patterns and statistical function for sampled time series of membrane potentials. Spiral wave can be found in the network under appropriate coupling intensity and these results could be associated with self-organization in the media.

2. Scheme description and model

The dynamical equations for the neuronal network in a two-dimensional square array is defined as follows

$$\begin{cases} \dfrac{dx_{ij}}{dt} = y_{ij} - ax_{ij}^3 + bx_{ij}^2 - z_{ij} + I_{ext} + \Gamma_{ij} \\ \dfrac{dy_{ij}}{dt} = c - dx_{ij}^2 - y_{ij} \\ \dfrac{dz_{ij}}{dt} = r[s(x_{ij} - \chi) - z_{ij}]; \end{cases} \tag{1a}$$

$$\Gamma_{ij} = \begin{cases} D(x_{i+1j} + x_{i-1j} + x_{ij+1} + x_{ij-1} - 4x_{ij}), & 1 < i,\ j < N \\ D(x_{ij+1} + x_{ij-1} - 2x_{ij}) + D(x_{i+1j} - x_{ij}), & i = 1 \\ D(x_{ij+1} + x_{ij-1} - 2x_{ij}) + D(x_{i-1\,j} - x_{ij}), & i = N \\ D(x_{i+1j} + x_{i-1j} - 2x_{ij}) + D(x_{ij+1} - x_{ij}), & j = 1 \\ D(x_{i+1j} + x_{i-1j} - 2x_{ij}) + D(x_{ij-1} - x_{ij}), & j = N \end{cases} \tag{1b}$$

where the variables x_{ij}, y_{ij}, z_{ij} represent the membrane potential, slow current associated with recovery variable and adaption current for neuron on node position (ij), respectively. D denotes the coupling intensity between nodes and I_{ext} defines the external forcing current. The square array consists of $N \times N$ nodes and neurons are placed on the nodes uniformly. According to Eq. (1b), we supposed that the nodes on the boundary can take coupling current from the inner nodes. For example, the node $(1, 3)$ can take any coupling current from node $(2, 3)$. For an isolate HR neuron mode, it can produce quiescent state, spiking, bursting and even chaotic state by increasing the external forcing current at fixed parameters $a = 1.0, b = 3.0, c = 1.0, d = 5, s = 4.0, r = 0.006, \chi = -1.56$. In fact, the mode of electrical activity is associated with energy storage and release; the Hamilton energy can be quickly released under bursting and chaotic states [26]. To discern the collective behaviors in electrical activities of neurons, a statistical factor of synchronization R is calculated in the