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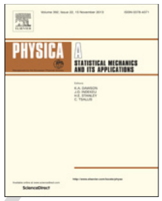

Q1 Determination of configuration matrix element and outer synchronization among networks with different topologies

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HIGHLIGHTS

- Outer synchronization among discrete networks with different topologies is researched.
- The control inputs of the networks are designed.
- The adaptive laws of configuration matrix element are obtained.

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ABSTRACT

In this paper, we research the outer synchronization among discrete networks with different topologies. Based on Lyapunov theorem, a novel synchronization technique is designed. Further, the control inputs of the networks and the adaptive laws of configuration matrix element are obtained. In the end, a numerical example is given to illustrate the effectiveness of the synchronization technique. It is found that the designed control input of the networks ensures the convergence of the errors among the networks to zero. And the designed adaptive law of configuration matrix element can replace effectively configuration matrix element in networks.

1. Introduction

In recent years, the dynamics of networks has been extensively researched. As a typical kind of dynamics, network synchronization has become a focal point due to the fact that it has many applications in various fields, such as the synchronous information exchange in the WWW internet, the synchronous transfer of digital or analog signals in the communication networks and so on [1–4]. So far, a variety of different synchronization phenomena, such as complete synchronization [5], generalized synchronization [6], phase synchronization [7], lag synchronization [8], anti-synchronization [9], etc., have been reported.

At present, the reported researches about network synchronization mainly focus on the situations that all nodes within a network achieve the coherent behavior, which is called inner synchronization. This technique was firstly proposed by Pecora and Carroll, who applied the Master Stability Function (MSF) method to determine the stability of the synchronous state in linear-coupled network [10]. After that, the research of inner synchronization of network has made significant advance, and some typical techniques of network synchronization are proposed, including adaptive method [11–13], pinning technique [14,15], impulse control [16], and so on.

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Note that the synchronization effect in reality exists not only among nodes within a network, but also among networks. For example, the infectious diseases including bird flu and SARS, are spread in different countries or populations. Information is transmitted synchronously among several local networks, etc. Therefore, the research on the synchronization among networks is very necessary and the synchronization among networks is called outer synchronization. Some typical works have been reported. Li et al. firstly investigated the synchronization problem between two networks with identical structure by using an open-plus-closed-loop controller [17]. Shortly after, Sun et al. studied projective synchronization in drive-response dynamical networks of partially linear systems with time-varying coupling delay [18]. Zhang et al. completed the function projective synchronization in drive-response networks with identical nodes based on Lyapunov stability theory [19]. Li and Xue researched the outer synchronization between two networks using arbitrary coupling strength based on the Lyapunov function method [20]. Wang et al. investigated the mixed outer synchronization between two complex networks with the same topological structure and time-varying coupling delay [21]. Du analyzed the problem of projective synchronization in drive-response dynamical networks based on the adaptive open-plus-closed-loop method [11]. Wu et al. achieved the anti-synchronization of two general complex dynamical networks with non-delayed and delayed coupling using pinning adaptive control method [22]. Yang and Jiang discussed the adaptive synchronization of complex networks with fractional-order derivatives and proposed an adaptive strategy to realize parameter identification in the process of synchronization [23]. These fruitful works establish a solid foundation for the theoretical research and practical application of outer synchronization techniques of networks.

As far as we know, most of outer synchronization techniques of network are mainly designed for the continuous network. But outer synchronization technique of multi-discrete networks with different topologies has not been reported. In this paper, we complete outer synchronization among discrete networks with different topologies through the design of a novel synchronization technique. The rest of the paper is organized as follows. In Section 2, the outer synchronization technique among networks is designed. A numerical example is illustrated in Section 3. Finally, a brief conclusion is drawn in Section 4.

2. Synchronization technique

Consider the following discrete chaotic system

$$x(n+1) = f(x(n)) \tag{1}$$

and rewrite Eq. (1) in the following form

$$x(n+1) = \xi x(n) + F(x(n)) \tag{2}$$

where n is discrete time series, $x(n) = (x_1(n), x_2(n), \ldots, x_l(n))^T \in R^l$ is the state variable of the system and ξ is the coefficient of linear term.

Consider M discrete networks with arbitrary different connections, in which each network consists of N identical nodes. The state equation of node i in network k is described by

$$x_i^k(n+1) = \xi x_i^k(n) + F(x_i^k(n)) + \mu_i \sum_{j=1}^{N} c_{ij}^k x_j^k(n) + u_i^k \tag{3}$$

where μ_i is the coupling strength of inner nodes in the network k and c_{ij}^k is the matrix element of the coupling matrix $C^k(c_{ij}^k)$ whose forms depend on the connection type of the network and express the topological structure of the networks. If there exists a link from node i to j $(i \neq j)$, then $c_{ij} \neq 0$; otherwise, $c_{ij} = 0$. u_i^k is the control input of the networks. It is worth noting that its diagonal elements meet $c_{ii} = -\sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij}$.

Denote errors among the network nodes

$$e_i^k(n) = x_i^k(n) - x_i^{k+1}(n) \quad (k = 1, 2, \ldots, M-1;\ i = 1, 2, \ldots, N) \tag{4}$$

then all networks will achieve synchronization if $e_i^k(n) \to 0$ as $n \to \infty$.

Further, we can obtain

$$\begin{aligned} e_i^k(n+1) &= x_i^k(n+1) - x_i^{k+1}(n+1) \\ &= \xi e_i^k(n) + \Delta F^k + \mu_i \sum_{j=1}^{N} c_{ij}^k x_j^k(n) - \mu_i \sum_{j=1}^{N} c_{ij}^{k+1} x_j^{k+1}(n) + u_i^k - u_i^{k+1} \end{aligned} \tag{5}$$

where $\Delta F^k = F(x_i^k(n)) - F(x_i^{k+1}(n))$.

The Lyapunov function of M networks is constructed as

$$V_n = \sum_{k=1}^{M-1} \sum_{i=1}^{N} \left| e_i^k(n) \right| + \sum_{k=1}^{M-1} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{\theta_{ij}^k} \left| c_{ij}^k - c_{ij}^{k+1} + a_{ij}^k(n) \right| \tag{6}$$

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