Physica A 462 (2016) 737–742

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

The equal load-sharing model of cascade failures in power grids

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HIGHLIGHTS

- We introduce a mean-field model for power grid failures.
- We show that the transition is first order.
- We show that the disorder can enlarge the transition region.

ARTICLE INFO

Article history:
Received 8 May 2015
Received in revised form 21 September 2015
Available online 11 June 2016

Keywords:
Complex networks
Power grid
Mean field models

ABSTRACT

Electric power-systems are one of the most important critical infrastructures. In recent years, they have been exposed to extreme stress due to the increasing power demand, the introduction of distributed renewable energy sources, and the development of extensive interconnections. We investigate the phenomenon of abrupt breakdown of an electric power-system under two scenarios: load growth (mimicking the ever-increasing customer demand) and power fluctuations (mimicking the effects of renewable sources). Our results indicate that increasing the system size causes breakdowns to become more abrupt; in fact, mapping the system to a solvable statistical-physics model indicates the occurrence of a first order transition in the large size limit. Such an enhancement for the systemic risk failures (black-outs) with increasing network size is an effect that should be considered in the current projects aiming to integrate national power-grids into "super-grids".

1. Introduction

The electrical power system (EPS) is crucial for the well-functioning of most critical infrastructures like telecommunications, banking systems, oil and gas pumping, water distribution [1]. Since their first appearance in 1881 at Godalming in England, EPSs have evolved into one of the most well engineered and robust network infrastructure; nevertheless power outages do occur with a likelihood larger than what would be naively expected. In particular, historical data show that outages' empirical probability distribution has fat tails [2,3], corresponding to a non-vanishing risk of system-wide failures (major outages or black-outs) causing disruption and economic damages.

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http://dx.doi.org/10.1016/j.physa.2016.06.034
0378-4371/

Black-outs often occur as a cascading sequence of failures and automatic disconnections triggered by an apparently minor initiating event; no two cascading outages are the same [4]. The mechanisms driving the tripping (disconnections) sequences are manifold and comprise the overloading of line and/or generators, frequency imbalances and transient currents. A black-out happens when the time-scales of automatic reactions are way too fast to correct the process by human intervention.

In order to predict and control the system, power engineers have developed and are refining sophisticated systems to simulate the full dynamics of whole power systems; moreover, distributed metering is going to furnish detailed data for the energy consumption at the household level [5]. Nevertheless, black-outs still occur and understanding the nature of such occurrences is still an open problem. A general question is whether such large outages can be partially due to emergent behaviour in the EPSs, i.e. whether is due to some critical point in the global behaviour of the system's components when considered as a whole: if this would be the case, increasing the accuracy of power systems' simulation would not result in better predictions of black-outs. The fact that EPSs are aggregations of large number of simple units makes them an ideal candidate to be a system exhibiting additional complexity as a whole beyond what is dictated by the simple sum of its parts.

To highlight the possibility of emergent behaviour, it is first necessary to simplify the system in order to understand the basic mechanisms that could drive systemic behaviour. We will hence introduce a simplified model of EPS that is amenable to both simulation on realistic grids and of a self-consistent analytical solution. We will then consider the behaviour of such a model under two kinds of stresses: (1) an increasing growth of the loads, mimicking the case of EPS that are operated to the limit of their capacities in order to maximize profits and (2) fluctuations in demands and generation, mimicking the effects of the steady penetrations of the erratic renewable sources. This latter case is of particular interest since the effects and consequences of introducing in the grids new erratic sources have not yet been fully understood.

2. Overload cascade model

A particular source of stress to EPS comes from the fact that adjustments in power generation are not real-time but follow fixed time schedules; for example, in Europe the production is fixed in advance the day before; the reaction time (apart from automatic controls/tripping) is generally much higher than the time of propagation of electrical perturbation in the system and is typically in the range of 15–30 min.

The tripping of lines and generators above their operating limits induced by automatic protective equipments is common to all kinds of cascading outages; while this process is intended to protect costly equipments from damage, it can potentially widen cascade failures [4].

Most of the cascade models for power grids are purely topological models based on the local redistribution of power loads upon failure [6–9] and disregard the long range nature of electricity. On one hand, a clear signature of the non-locality of power outages can be found in real data: this is, for example, the case of the tripping sequence of the Western Interconnection WSCC system disturbances in July 2–3, 1996 [10], where the occurrence of subsequent failures in far away lines can be observed. On the other hand, it is possible to have simple yet realistic models respecting Kirchoff laws, like the overload cascade model (OCM) introduced by Pahwa et al. [11].

In Pahwa's model, an initial power flow configuration is calculated using the DC power flow model [12]; link capacities (i.e. the maximum of power-flow before a link failure) is proportional to the initial flow on the branch and is usually assumed to be 10% higher than the initial flow. Notice that this condition is equivalent to the normal bounds assumed for the validity of the DC power flow model. To understand what happens when the system is subject to stress, a new configuration of loads is assumed and power flows recalculated. If the load on a line goes beyond its capacity, the line trips (disconnects) and power flows are recalculated on the new topology (i.e. the grid MINUS the tripped lines). Such procedure is repeated until convergence. Notice that OCM is a simplified version of the dynamic OPA model introduced by Carreras et al. [13] that is also based on DC power flow equations but uses linear optimization for generation dispatch after each link failure step. Further simulation evidence of the nature of a phase transition associated to load increases can be found in Refs. [14,15].

3. Equal load-sharing model

To understand the possible mean-field approximation for the OCM model, we have first to describe in detail its form. The OCM model, as several similar ones, is based on DC power flow equations. In the DC power flow, the grid is described by matrix Y_{ij} representing the admittance of the M edges (branches) ij among N nodes (buses). The power injected (generators) or absorbed (loads) on the ith bus is described by the vector P_i. The variables describing the electric state of the systems are the phase angles θ_i at the buses that are linearly related to the injected powers:

$$\mathcal{L}\vec{\theta} = \vec{P} \tag{1}$$

where $\mathcal{L}$ is the Laplacian associated with the admittance matrix Y. Power flows along branches are proportional to the phase-angle differences $\theta_{ij} = \theta_i - \theta_j$.

Since Eq. (1) is in the form of a Laplacian, the interactions among phase angles θ_i is long range; in fact, similar equations hold to describe resistor network models [16] or simplified fracture models [17]. Hence, when an edge breaks down in such kind of systems, the stress it was carrying (in our case the power flow) will redistribute among all the remaining edges. To capture the above effect, we can hence develop a mean-field model of the OCM by assuming that the flow carried by a