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Q1 Optimal weighted suprathreshold stochastic resonance with multigroup saturating sensors

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HIGHLIGHTS

- A general noise-enhancement mechanism of the optimal weighted suprathreshold stochastic resonance.
- Optimal weighted decoding scheme for multigroup sensor arrays.
- Superiority of the optimal weighted sensors over the unweighted sensors in MSE distortion.

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ABSTRACT

Suprathreshold stochastic resonance (SSR) describes the noise-enhanced effect that occurs, not in a single element, but rather in an array of nonlinear elements when the signal is no longer subthreshold. Within the context of SSR, we investigate the optimization problem of signal recovery through an array of saturating sensors where the response of each element can be optimally weighted prior to summation, with a performance measure of mean square error (MSE). We consider groups of sensors. Individual sensors within each group have identical parameters, but each group has distinct parameters. We find that the optimally weighting array always provides a lower MSE in comparison with the unweighted array for weak and moderate noise intensities. Moreover, as the slope parameter of sensors increases, the MSE superiority of the optimally weighting array shows a peak, and then tends to a fixed value. These results indicate that SSR with optimal weights, as a general mechanism of enhancement by noise, is of potential interest to signal recovery.

1. Introduction

Stochastic resonance (SR) essentially represents a class of phenomenon where, for certain types of nonlinear coupling between signal and noise, the presence or the addition of noise provides an improved performance over the absence of noise [1–5]. This counter-intuitive effect was first introduced in the field of meteorology [1], and it has gradually been observed in a wide variety of fields, for instance, physics, biology and electronic engineering [2–23]. Initial studies of SR mainly focused on dynamical systems with a nonlinearity due to a simple threshold operation or a potential barrier [1–23], where the enhanced response of a weak signal results from noise-assisted threshold or barrier crossings.

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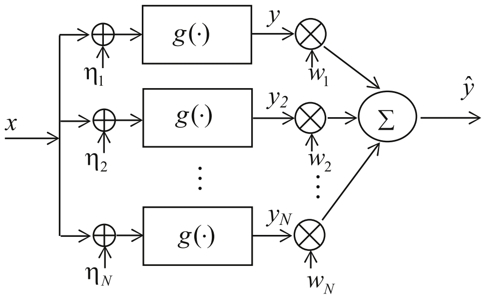

Fig. 1. Weighted summing array of N identical noisy nonlinear elements, $g(\cdot)$. Each of these N elements operates on a common signal x subject to additive noise η_i. The output of each individual element y_i is multiplied by the weighting coefficient w_i, and the overall output $\hat{y} = \sum_{i=1}^{N} w_i y_i$.

With a growing interest in SR, the threshold-free or barrier-free nonlinear system is also found to demonstrate noise-enhanced signal transmission effects [24–28]. Especially, some nonlinearities with saturation have been observed to exhibit SR with improved signal-to-noise ratio, cross-correlation and mutual information [27,28]. Furthermore, another distinct SR mechanism was explored in coupled or uncoupled parallel arrays of nonlinear systems which significantly extended the concept of SR to broader conditions [29–45]. These related results show that the summed response of parallel nonlinear elements with uncorrelated noise can be more efficient, under certain performance measures, than using a single nonlinearity with no noise. Of course, each element in these parallel arrays can be threshold-free [41–45] or can perform a thresholding operation [29–40], and in the latter case, suprathreshold stochastic resonance (SSR) occurs for a summing network of threshold comparators [30–32].

For the case of arrays of identical threshold comparators, SSR can be also described in terms of stochastic signal quantization [36–39]. In line with this concept, McDonnell et al. have examined SSR with emphasis on finding the optimality of the quantization, in terms of lossy source coding and quantization theory, by using the mean square error (MSE) as the measure of distortion [36–38]. Recently, we investigated the decoding of a quantized signal and proposed an optimal weighted decoding scheme [39]. Our previous study showed that, for particular noise levels and threshold value distributions, the performance of optimally weighting the quantizer responses is superior to the original unweighted array [39].

However, the previous work [39] restricts the optimal weighted decoding scheme only to threshold nonlinearities. In this paper, we will generalize the optimal weighted decoding scheme to arrays composed of arbitrary nonlinearities, and derive the expression of the decoding output for multigroup parameter settings. Specifically, we apply the decoding approach to arrays of saturating sensors. By dividing the nonlinear elements into different sized groups, we compare optimally weighting the element responses with the original unweighted arrays. The results show that, with regular intervals of shifted parameters in multigroups, the MSE performance of optimal weighted decoding is superior to that of the original unweighted arrays for weak and moderate noise intensities. Moreover, this superiority of the MSE distortion becomes more evident with increasing group size. In addition, as the slope parameter varies, the MSE difference of two different decoding schemes reaches a maximum, and then decays to a fixed value. Finally, at a given noise level and for the group size of two, we also compare two different decoding schemes in the case of optimized shifted parameters within each group.

2. Model and method

We consider a weighted summing array of N noisy nonlinear elements receiving an input random signal $x(t)$ with standard deviation σ_x, as shown in Fig. 1. Each element of the array is endowed with the same input–output characteristic, modeled by the static (memoryless) function g. The ith nonlinear element is subject to independent and identically distributed (i.i.d.) additive noise component η_i with standard deviation σ_η, which is independent of the signal $x(t)$. Accordingly, each element produces the output signal $y_i(t) = \mathrm{g}[x(t) + \eta_i(t)]$. The output signal $y_i(t)$ is multiplied by the weighting coefficient w_i ($w_i \in \Re$), so as to yield the weighted output $w_i y_i$. Then, all weighted outputs are summed to give the overall output of array $\hat{y} = \sum_{i=1}^{N} w_i y_i$.

When all weighting coefficients w_i are equal to unity, the decoding method is performed by weighting after summation for $i = 1, 2, \ldots, N$. When the weighting coefficient w_i is arbitrarily chosen, the decoding function is performed by weighting before summation. It is known that if the reconstruction points are linearly spaced, then the optimal reconstruction points are given by Wiener decoding [46] which is carried out by weighting after summation. For the case of $\mathrm{E}[x] = 0$, the reconstructed value $\hat{y}_{\mathrm{w}}$ with Wiener linear decoding is expressed by [38,46]

$$\hat{y}_{\mathrm{w}} = \frac{\mathrm{E}[xy]}{\mathrm{var}[y]}(y - \mathrm{E}[y]), \quad (1)$$

where $y = \sum_{i=1}^{N} y_i$ represents the unweighted sum of the array response and $\mathrm{var}[y]$ is the variance of y.

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