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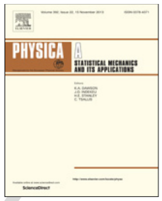

Theoretical and experimental implementation of vibrational resonance in an array of hard limiters

Yuhao Ren, Fabing Duan *

Institute of Complexity Science, Qingdao University, Qingdao 266071, China

HIGHLIGHTS

- Vibrational resonance effect in a parallel array of hard limiters.
- The extreme values of the signal-to-noise ratio gain exhibit a bifurcation behavior.
- A tractable experimental realization of an infinite array by finite arrays.

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ABSTRACT

We report that the output signal-to-noise ratio (SNR) of a parallel array of hard limiters can be maximized at an optimal high-frequency vibration amplitude, i.e. the vibrational resonance (VR) effect. As the external noise shape parameter varies, the bifurcation mode of maximal SNR gain is found, and the upper limit of SNR gain is discussed. We theoretically demonstrate a tractable realization of an infinite array approached by a finite array of two hard limiters, and design an electronic circuit experiment to verify the feasibility of this effective method. These results indicate the potential applications of vibrational devices to array signal processing.

1. Introduction

In the last three decades, the stochastic resonance phenomenon put forward by Benzi et al. [1] became a hot topic in the field of nonlinear science. It is now well known that an optimal amount of noise can enhance the responses of certain nonlinear systems to a weak input signal [2–8]. The resonance mode is really manifold. For instance, Vilar and Rubí observed the stochastic multiresonance phenomenon [9] that is subsequently verified in oscillators [10–12], threshold-crossing systems [13], and laser devices [14]. More importantly, Landa and McClintock found a similar effect referred to as vibrational resonance (VR), whereby the role of noise is replaced by a high-frequency periodic force [15]. Since then, a series of studies of VR are carried out in electronic circuits [16,17], excitable neurons [18], optical devices [19], bistable oscillators [20–23], and dynamical oscillators [24–26]. Interestingly, it was demonstrated in a bistable optical device that the signal-to-noise ratio (SNR) due to VR is always higher than the one obtained through the use of the stochastic resonance effect [27]. Recently, inspired by the stochastic resonance in arrays [28–33], we inject sinusoidal vibrations with different high frequencies into an array of nonlinear devices, and confirm the VR effect of the output–input SNR gain [34].

We note that the stochastic resonance method, i.e. tuning the internal noise components in an array, might be not easy to put into effect, since the noise type or level is difficult to control. Then, due to the easy implementation of high-frequency sinusoidal vibrations, we further investigate the VR effect in a parallel array of hard limiters in this paper. For

* Corresponding author.
E-mail address: fabing.duan@gmail.com (F. Duan).

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a weak sinusoidal signal buried in a given external noise, we can improve the output–input SNR gain by optimizing the amplitude of sinusoidal vibrations and increasing the array size. When the external noise type varies, the extreme values of SNR gain exhibit a bifurcation behavior with its critical point corresponding to the Gaussian distribution. Moreover, we find the maximal SNR gain achieved by the mechanism of VR in an infinite array is still bounded by the upper limit of Fisher information. It is proved that an infinite array of hard limiters can be closely approached by finite arrays of moderate size, wherein the maximal SNR gain can be effectively obtained. Then, this tractable method is experimentally demonstrated in an electronic circuit of hard limiters. We argue that the easily implementable feature of sinusoidal vibrations, as well as the achievable maximum of SNR gain of an infinite array, indicates a preferable strategy for processing weak signals via the VR mechanism.

2. Model and measure

Consider a signal-plus-noise mixture of $s(t)+\xi(t)$, where $s(t)$ is deterministic with a maximal amplitude $A\,(0<|s(t)|\leq A)$ and period T, and the external white noise $\xi(t)$ is stationary, independent with $s(t)$, with the probability density function (PDF) $f_\xi(u)=\mathrm{d}F_\xi(u)/\mathrm{d}u$ and a root-mean-square (RMS) amplitude σ_ξ. The input SNR for $s(t)+\xi(t)$ can be defined as the power contained in the spectral line $1/T$ divided by the power contained in the noise background in a small frequency bin ΔB around $1/T$ [5], this is

$$R_{\text{in}}=\frac{|\langle s(t)\exp[-\mathrm{i}2\pi t/T]\rangle|^2}{\sigma_\xi^2\Delta t\Delta B}, \tag{1}$$

where Δt indicates the time resolution in a discrete-time implementation and the temporal average operator $\langle\cdots\rangle=\frac{1}{T}\int_0^T\cdots\mathrm{d}t$. Next, we apply this mixture to a memoryless system g with its output

$$y(t)=g[s(t)+\xi(t)]. \tag{2}$$

Similarly, based on the cyclostationarity property of $y(t)$, the output SNR for $y(t)$ is given by [5]

$$R_{\text{out}}=\frac{|\langle \mathrm{E}[y(t)]\exp[-\mathrm{i}2\pi t/T]\rangle|^2}{\langle\mathrm{var}[y(t)]\rangle\Delta t\Delta B}, \tag{3}$$

with nonstationary expectation $\mathrm{E}[y(t)]$ and nonstationary variance $\mathrm{var}[y(t)]$.

Assume $s(t)$ is weak ($A\to 0$), and make a Taylor expansion of g around ξ at a fixed time t as $y(t)=g[\xi+s(t)]\approx g(\xi)+s(t)g'(\xi)$ with the derivative $g'(\xi)=\mathrm{d}g(\xi)/\mathrm{d}\xi$. Here, the Taylor expansion of g is up to first order in the limit of a small signal $s(t)$. We further assume that g has zero mean and finite variance under f_ξ, i.e. $\mathrm{E}[g(\xi)]=\int_{-\infty}^{\infty}g(\xi)f_\xi(\xi)\mathrm{d}\xi=0$ and $\mathrm{E}[g^2(\xi)]<\infty$. Then, we approximate $\mathrm{E}[y(t)]\approx s(t)\mathrm{E}[g'(\xi)]$ and $\mathrm{var}[y(t)]\approx\mathrm{E}[g^2(\xi)]$ [35]. Substituting $\mathrm{E}[y(t)]$ and $\mathrm{var}[y(t)]$ into Eq. (3), we have

$$R_{\text{out}}\approx\frac{|\langle s(t)\exp[-\mathrm{i}2\pi t/T_s]\rangle|^2}{\Delta B\Delta t}\frac{\mathrm{E}^2[g'(\xi)]}{\mathrm{E}[g^2(\xi)]}\approx R_{\text{in}}\sigma_\xi^2\frac{\mathrm{E}^2[g'(\xi)]}{\mathrm{E}[g^2(\xi)]}. \tag{4}$$

Then, using the Schwarz inequality, the output–input SNR gain is bounded by [35]

$$G=\frac{R_{\text{out}}}{R_{\text{in}}}\approx\sigma_\xi^2\frac{\mathrm{E}^2[g'(\xi)]}{\mathrm{E}[g^2(\xi)]}\leq\sigma_\xi^2\mathrm{E}\left[\frac{f_\xi'^2(\xi)}{f_\xi^2(\xi)}\right]=\sigma_\xi^2 I(f_\xi)=I(f_{\xi_n}), \tag{5}$$

where equality achieved when g takes the locally optimal system $g_{\text{opt}}(\xi)=Cf_\xi'(\xi)/f_\xi(\xi)$ for a constant C and the Fisher information $I(f_\xi)=\mathrm{E}\left[f_\xi'^2(\xi)/f_\xi^2(\xi)\right]$. Here, the scaled noise $\xi(t)=\sigma_\xi\xi_n(t)$ has PDF $f_\xi(\xi)=f_{\xi_n}(\xi/\sigma_\xi)/\sigma_\xi$, and the standardized noise PDF f_{ξ_n} is with unity variance $\sigma_{\xi_n}^2=1$. In some situations, the optimal nonlinearity g_{opt} is not easily constructed or may be unknown [36,37]. This is because that the noise distribution f_ξ may be unavailable, or does not exist. For instance, an α-stable random variable is defined by the characteristic function, and the corresponding density function is not explicitly known [38]. Therefore, in practice, suboptimal processors are often employed, which comprises a simple and feasible characteristic in Eq. (2), e.g. an *ad hoc* piecewise function [36,37].

Aiming to improve the output SNR, the stochastic resonance method adds extra noise into suboptimal processors. However, for a given noisy input $s(t)+\xi(t)$, the addition of more noise may only degrade the performance of a single system of Eq. (2). This is because that, for a given system, the optimal noise level might be less than the external noise RMS amplitude σ_ξ. Thus, in order to further exploit the benefit of noise, a parallel array of N identical nonlinear subsystems subjected to a common given noisy signal is adopted, wherein each subsystem of array is driven by the mutually independent internal noise components. Upon increasing the array size and optimally tuning the internal noise level, the performance
Q3 enhancement of an array can be achieved [28–34,38].

We note that the internal noise type or level might be difficult to control. Then, we adopt the method of VR, and replace the internal noise components in an array by high-frequency vibrations

$$\eta_n(t)=A_\eta\sin(2\pi f_n t), \tag{6}$$

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