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An alternative method for modeling the size distribution of top wealth

Yuanjun Wang [a,*], Shibing You [b]

[a] Department of Economics, Wuhan University of Technology, Wuhan 430000, PR China

[b] Department of Economics, Wuhan University, Wuhan 430072, PR China

HIGHLIGHTS

- We suggest new Lorenz curve models for describing the size distribution of top wealth.
- The new models have a decreasing density.
- The new models can provide better approximation than the classical Pareto distribution.
- We show that some popular method for estimating the parameters of the Pareto distribution can provide implausible results.

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ABSTRACT

The Pareto distribution has been widely applied in modeling the distribution of wealth, as well as top incomes, cities and firms. However, recent evidence has shown that the Pareto distribution is not consistent with many situations in which it was previously considered applicable. We propose an alternative method for estimating the upper tail distribution of wealth and suggest a new Lorenz curve for building models to provide such estimates. Applying our new models to the Forbes World's Billionaire Lists, we show that they significantly outperform the Pareto Lorenz curve as well as some other popular alternatives.

1. Introduction

Many empirical distributions found in various disciplines exhibit power-law behavior. The fields relevant to economics include the size distributions of cities, firms, top incomes and top wealth. As pointed out by Ogwang [1], the power-law model has probably received the most attention in income and wealth distribution studies, in which it has become an accepted method for modeling top incomes. Recently, a rapidly increasing literature concentrates on measuring the income share of top income groups, see, for example, Atkinson [2], Atkinson and Piketty [3,4], Banerjee and Piketty [5] and Piketty [6], in which the top shares are estimated with a Pareto distribution.[1]

The research of Atkinson and others is a continuation and generalization of the venture what Kuznets did in the 1950s (see Kuznets [9]), as pointed out by Atkinson and Piketty [3]. Kuznets proposed an inversed-U hypothesis, which describes

* Corresponding author.
E-mail address: wyj_carmen@whut.edu.cn (Y. Wang).

[1] The income share s of the very top group can be used as an inequality index, since the Gini index, which is the must widely used income inequality measure, can be expressed approximately as $s + (1 - s)G$, provided the population share of the top group is very small, where G is the Gini index for the rest of the population (see Alvaredo [7]; Atkinson and Piketty [3, p. 19]). For a further discussion of the relationship between the top share and other often used inequality indices, see Leigh [8].

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the relationship between economic development and income inequality (see Kuznets [10]). According to the theory, income inequality should first rise with industrialization and urbanization, and then decline as more and more workers join the high productivity sectors of the economy. Because of its immense theoretical and policy implications, enormous related literature has emerged since the emergence of the theory. With the power-law a main model and the top share an inequality measure, using the long-run tax data dating from as early as the beginning of the last century, Atkinson and others examine the income inequality trends for a number of countries. Apart from many other significances of the research, they raise serious doubts about the hypothesis by presenting top share time series with a length unusual in economics literature (Atkinson and Piketty [3, p. 9]).

In spite of the success of the power-law in measuring the top income share, controversy still exists over the best way to model the size distributions of cities, firms and top wealth. While Jayadev [11], Klass et al. [12], Levy and Solomon [13], Sinha [14], among others, find that the size distribution of top wealth follows a power-law, using the framework developed by Clauset et al. [15], Brzezinski [16] finds that the power-law model is consistent with the data for only 35% of various Forbes lists. In studies of city or firm size distributions, the power-law or a special case called Zipf-law with power exponent of 1, has been tested in a number of studies, see, for example, Axtell [17], Fujiwara et al. [18], Gaffeo et al. [19], Gangopadhyay and Basu [20], Okuyama et al. [21], among others. However, Bee et al. [22] find that the size distribution of US cities do not follow the power-law. Anderson and Ge [23] find that the log normal distribution is a better model to describe Chinese city data. Marsili [24] shows that the power-law can only fit well at the extreme upper tail of the size distribution of Dutch manufacturing firms.

Many Lorenz curve models have been developed in the economics literature (see, for example, Ogwang and Rao [25], Sarabia et al. [26], Wang et al. [27]) for estimating the income distribution of an entire country or an area from grouped data. We suggest using this approach to model the size distribution of the upper tail of wealth. We also propose a new one-parameter Lorenz model which is proven useful in building new models for approximating the size data of wealth. To our knowledge, no other studies have used this approach to analyze top wealth. This approach is also useful when estimating size distributions of top incomes, cities or firms. Special features of our method include: its associated density is decreasing, it includes the power-law model as a particular case, and it outperforms the power-law model in our illustrative tests. Our tests also demonstrate that traditional procedures for estimating the power-law parameters may produce unsatisfactory results.

The paper is arranged as follows. The Lorenz curve method, as well as our new Lorenz curve model, is described in the next section. Our models are applied to several datasets from Forbes World Billionaire List (WBL) in Section 3, thus demonstrating the usefulness of the Lorenz model approach and our new Lorenz curve model for modeling the size distribution of the upper tail of wealth. Section 4 presents our conclusion.

2. Lorenz curve method for approximating the size distribution of top wealth

A Lorenz curve model $L(p)$ is a parametric Lorenz curve, where $p = F(x)$ is the cumulative density function (cdf) of the underlying income distribution. $L(p)$ is the income share of individuals with incomes less than x, which is defined as

$$L(p) = \frac{1}{\mu}\int_{x_0}^{x} sf(s)\mathrm{d}s, \quad p = F(x)$$

where $f(x)$ is the associated density function (df), μ is the average of the distribution and $x_0 \geq 0$ is a certain lower limit. $L(p)$ should satisfy

$$L(0) = 0, \qquad L(1) = 1, \qquad L'(p) \geq 0, \qquad L''(p) \geq 0.$$

Note we have $L'(p) = \frac{\mathrm{d}L(p)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}p} = \frac{x}{\mu}$. The df is

$$f(x) = \frac{1}{\mu L''(p)}, \quad p = F(x). \tag{2.1}$$

For example

$$L_\beta(p) = 1 - (1-p)^\beta, \quad \beta \in (0, 1], \tag{2.2}$$

is the Lorenz curve associated with the Pareto distribution in the literature. From the relationship given in (2.1), it can be verified that the associated df of $L_\beta(p)$ is

$$f_P(x) = \frac{1}{1-\beta} x_0^{\frac{1}{1-\beta}} x^{-\frac{2-\beta}{1-\beta}}, \quad x \geq x_0,\ \beta \in (0, 1) \tag{2.3}$$

with $x_0 = \mu\beta$.

Let $b = 1/(1-\beta)$, $f_P(x)$ can be rewritten as the usual form

$$f_P(x) = bx_0^b x^{-b-1}, \quad x \geq x_0,\ b > 1, \tag{2.4}$$