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Q1
Correlations of multiscale entropy in the FX market

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HIGHLIGHTS

- FX market dynamics is more complex and disordered for smaller time scales.
- Long term trends show periodic cycles of regular and irregular fluctuations.
- Price variations are weakly correlated with strong correlations at biweekly trends.

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ABSTRACT

The regularity of price fluctuations in exchange rates plays a crucial role in FX market dynamics. Distinct variations in regularity arise from economic, social and political events, such as interday trading and financial crisis. This paper applies a multiscale time-dependent entropy method on thirty-three exchange rates to analyze price fluctuations in the FX. Correlation matrices of entropy values, termed entropic correlations, are in turn used to describe global behavior of the market. Empirical results suggest a weakly correlated market with pronounced collective behavior at bi-weekly trends. Correlations arise from cycles of low and high regularity in long-term trends. Eigenvalues of the correlation matrix also indicate a dominant European market, followed by shifting American, Asian, African, and Pacific influences. As a result, we find that entropy is a powerful tool for extracting important information from the FX market.

1. Introduction

The foreign exchange market (FOREX, FX) has a huge impact on world economy, affecting the personal fortunes of billions. It manages the trading of currencies, which amounts to trillions of dollars being exchanged every day, and has direct influence on all other financial markets, since any price can be expressed in terms of a currency [1]. However, large volumes and decentralization (transactions take place all over the world) make it extremely difficult to control relative values between different currencies, i.e. exchange rates. The complex nature of FX market dynamics arises from the absence of an independent frame of reference for currency pricing, i.e. any currency must be expressed in terms of a *base currency*, and from its sensitivity to interactions with all other financial markets. An important area of research involves understanding the behavior and interactions of currencies which lead to such market dynamics.

Exchange rates can be typically expressed in terms of currency fluctuations, characterized by short and long-term trends. Such fluctuations often have complex and nonlinear interactions, and result in highly nontrivial structures, which makes

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it hard to extract any useful information. As a measure of disorder, entropy has become a powerful tool for quantifying the diversity (richness) and irregularity of exchange rate fluctuations. Its ability to extract information that is otherwise obscured by the complex nature of financial series [2,3], distinguishes entropy from many other forms of statistical measures. In particular, entropic analysis has been extensively used to describe phenomena arising in foreign exchange rates [4–8].

While entropy allows us to study disorder in currency fluctuations, it has no mechanism for comparing the unpredictability (or even complexity) between different exchange rates. However, such relations could provide valuable insights for understanding FX market dynamics. Correlation seems the ideal measure to tackle this problem because it can identify linear relationships between currencies. In particular, correlation matrices are very useful for comparing and analyzing pairwise interactions in the system.

In this paper we analyze currency fluctuations using a multiscale time-dependent entropy scheme which captures all the possible patterns on different time scales, and permits the characterization of both short and long-term financial trends. Correlations are used to study the market dynamics arising from interactions between different exchange rates, by quantifying the linear relationships of their entropic fluctuations. The evolution of entropy is measured for thirty three foreign exchange rates in a period ranging from 1971 to 2014. Correlations of entropy values, also termed entropic correlations, are investigated at different time scales and compared between different geopolitical landscapes.

The rest of this paper is organized as follows. Section 2 introduces the methodology related to entropy and correlation. Section 3 describes the experimental data and presents the empirical results. Section 4 draws the conclusions.

2. Methodology

2.1. Approximate entropy

A direct application of entropy assumes the data series is infinitely long with infinite accuracy and resolution [9]. Pincus [10] introduced approximate entropy to alleviate such demands by modifying the Kolmogorov–Sinai entropy, an exact regularity statistic. Approximate entropy quantifies the unpredictability (irregularity) within a time series of finite length, that is, series with high entropy are characterized by higher frequencies of irregular fluctuations than series with low entropy. Scale invariant and model independent, the measure is also complementary to other statistical methods such as spectral and autocorrelation analysis [11].

For a given financial series, the idea of using approximate entropy is to analyze the underlying fluctuations responsible for market dynamics. This measure represents the irregularity of prices in financial systems, where values of low entropy indicate ordered price fluctuations and random behavior is approximated for high entropy values. Approximate entropy has already proven successful in an array of applications including [12–14] market stability and efficiency [9,15], foreign exchange rates [8],commodity futures [16], crude oil markets [17], and stock prices [18].

An algorithm for calculating approximate entropy can be described as follows. Given a time series $Z = z_1, \ldots, z_N$, two m-dimensional sequence vectors, $x_i = z_i, \ldots, z_{i+m-1}$ and $y_j = z_j, \ldots, z_{j+m-1}$ for $1 \le i, j \le N - m + 1$, are selected and their distance is calculated as

$$d_{i,j}^{x,y} = \max_{k=0,\ldots,m-1} \left\{ \left| x_{i+k} - y_{j+k} \right| \right\} \tag{1}$$

where x_i and y_j are called similar if $d_{i,j}^{x,y}$ is smaller than a specified tolerance r. For each of the $N - m + 1$ elements, n_i is calculated as the total number of vectors y_j similar to x_i. The relative frequency (regularity) of finding similar vectors can then be expressed as $f_i(r) = \frac{n_i}{N-m+1}$, where $N - m + 1$ denotes the number of possible vectors. Next, one calculates the average value of the logarithm of $f_i(r)$, i.e. the average regularity

$$\phi^m(r) = \frac{1}{N - m + 1} \sum_{i=1}^{N-m+1} \ln f_i(r) \tag{2}$$

from which the approximate entropy can be estimated by

$$ApEn(m, r, N) = \phi^m(r) - \phi^{m+1}(r) \tag{3}$$

where m and r are variable parameters. Notice that $ApEn$ compares the average regularity of neighboring dimensions m and $m+1$; it represents the tendency that two patterns of fluctuations will retain their similarity at larger scales. In other words, lower values of $ApEn$ reflect more regular time series, while higher values correspond to less predictable (more complex) fluctuations [9].

2.2. Multiscale entropy

Entropy is well known to be scale-dependent, a pattern can appear more regular or less regular (more complex) depending on the time scale. In financial applications, this scale dependence manifests itself as short and long-term trends—fluctuations are more complex at smaller time scales due to unpredictability of the market, while larger time scales are