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Determination of zero-coupon and spot rates from treasury data by maximum entropy methods

Henryk Gzyl [a,*], Silvia Mayoral [b]

[a] Centro de Finanzas, IESA, Ave. Iesa, San Bernardino, Caracas 1010, Venezuela

[b] Business Administration, Univ. Carlos III de Madrid, C/Madrid 126, 28903, Getafe. Madrid, Spain

HIGHLIGHTS

- We present a model free method for determining spot prices from bond data.
- The method is based upon the method of maximum entropy.
- The method admits extensions to solve problems with errors in the data.
- Mispriced bonds can be regarded as inverse problems with errors in the data.
- The method can be easily extended to use bid–ask prices as inputs.

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ABSTRACT

An interesting and important inverse problem in finance consists of the determination of spot rates or prices of the zero coupon bonds, when the only information available consists of the prices of a few coupon bonds. A variety of methods have been proposed to deal with this problem. Here we present variants of a non-parametric method to treat with such problems, which neither imposes an analytic form on the rates or bond prices, nor imposes a model for the (random) evolution of the yields. The procedure consists of transforming the problem of the determination of the prices of the zero coupon bonds into a linear inverse problem with convex constraints, and then applying the method of maximum entropy in the mean. This method is flexible enough to provide a possible solution to a mispricing problem.

1. Introduction

The knowledge of the prices of the zero coupon bonds is essential for the valuation of numerous financial products, the cash flow associated with any project, or for the determination of forward spot rates. Even when trustworthy models of the short or of the spot rates are available, their calibration may rely on the knowledge of the price of zero coupon bonds.

Apart from the stochastic models, which propose a dynamics for either the short or the spot rates, two widely used types of methods are the parametric methods and the interpolation methods. In the class of parametric methods, a model is proposed for the spot rate for each maturity, and then the parameters are determined by requiring that the price of coupon bonds are as close as possible to the market prices. A simple version of this type of method is mentioned in the textbooks by Rupertt's [1] or Elliott and van der Hoek's [2]. A widely used model consist of the

* Corresponding author.
E-mail addresses: henryk.gzyl@iesa.edu.ve (H. Gzyl), smayoral@emp.uc3m.es (S. Mayoral).

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method originally proposed by Nelson–Siegel [3], Waggoner [5] or its extension by Svensson [4]. Variations on the theme of these proposals are gathered in the BIS paper No. 25 [6]. This is a collection of papers describing the procedure employed by several central banks in the late 1990s to deal with the problem that we are addressing here. A more recent comparative review is contained in the report by de Pooter [7], in which the performance of the various extensions of the Nelson–Siegel model are considered. We should mention as well that the Nelson–Siegel model has been used as input for neural network methodology to determine the relevant parameters by Rosadi and Dewi [8]. Consider as well the relatively recent work by Diebold, Rudebusch and Aruoba [9], Diebold, Li and Yue [10] and Christensen, Diebold and Rudebusch [11] in which different extensions of the Nelson–Siegel model are examined. In the first of these, the authors provide an extension of the Nelson–Siegel model, in which the coefficients become time dependent functions (called latent factors) that can be used to provide an even larger class of models. In the second paper the authors show how to extend the Nelson–Siegel–Svensson set up to render it arbitrage free. As representative of the class of interpolation methods, consider the cubic spline interpolation proposed by McCulloch [12] and revised in Refs. [13,14]. This consists in interpolating the zero coupon price curve by means of cubic polynomials. The continuity conditions at each node are combined with the coupon bond prices to obtain a linear system that is solved for the coefficients of the approximant between nodes. For more on this method consider Johnston [15] and Shea [16,17]. For an earlier variation on the interpolation theme, see Ref. [18], where an interpolation procedure using cubic exponential splines was proposed to fit the interest rate term structure. Still another class of interpolation methods is described in Ref. [19]. Of these we review the B-spline interpolation method below. We mention that the MATLAB codes for the implementation of this method by Mazzarina [20].

More or less recently Munk [21], Brigo and Mercurio [22] and Rebonato [23]present a totally different collection of approaches to the modeling and calibration of the term structure of interest rates. See as well the approach of Diebold et al. [24], Ioffe and Prisman [25] and Chib and Ergashev [26] for an attempt to understand interest rates in a wider context.

As far as procedures that use the maximum entropy methods for interest rate modeling, we should mention Gulko [27] and Brody and Hughston [28]. Interesting as these two papers are, they deal with problems and techniques not related to the problem that we are interested in. As of the present date, there does not seem to exist work on application of the maximum entropy method to interest rate calibration problems. Nevertheless, there exists a large number of papers that deal with the maximum entropy for volatility calibration, to determine risk neutral densities and risk aversion measures. A short sample of papers applying maximum entropy comprises Rogers [29], Avellaneda et al. [30], Buchen and Kelly [31], Shiriayev [32], Bülhmann et al. [33], Borwein et al. [34], Frittelli [35], Goll and Rüschendorf [36], Gzyl and Mayoral [37], Neri and Schneider [38]. See as well the recent review by Zhong et al. [39] where many applications on the method of maximum entropy in finance are reviewed. To finish, long list of applications of the method of maximum entropy in many different fields appears in Ref. [40].

The method that we propose to deal with the bond price term structure is called the method of maximum entropy in the mean, which builds upon the standard maximum entropy method. This method designed to deal with linear inverse problems with convex constraints. From an abstract point of view, one can summarize the method saying that to systematically pick up a point in a convex set, just maximize a concave function defined over the given set. For full details, applications and bibliographical sources, we refer the reader to Dacunha-Castelle and Gamboa [41], Marechal [42] Gzyl or to Velásquez [43].

The standard maximum entropy method seems to have first been proposed by Jaynes [44] as a variational method for finding equilibrium distributions in statistical physics. The physical intuition behind it goes back to the foundations of thermodynamics, but in its present formulation, the idea behind it has existed in some form or another for some time. In actuarial sciences the method is called Esscher transform and in statistics it is called exponential tilting.

In Section 2 of the paper we state the basic equations that relate the zero coupon prices to the market prices of available bonds, and then transform them to be able to apply the maximum entropy method to the problem. Section 3 is devoted to the maxent representation of the solutions to the problem. There are several possible approaches depending on how one thinks about the mathematical statement of the problem (completed in (2.5)). First we consider a solution using the standard maximum entropy method, the two variations on the theme of the maximum entropy of the mean. The second one would be useful if one had bounds on the underlying term structure of interest rates.

It is in Section 3.4 where the real power of the method is displayed. Here we present an extension of the method developed in Section 3.2 to deal with the case of the bonds which we think that might be mispriced or inconsistently priced.

The material presented in the different sub-sections can be found in extenso in the volume by Gzyl and Velásquez. In Section 3 we recall the bare minimum necessary to state a possible solution to (2.5).

In Section 4 we rapidly review three standard methods, the cubic spline interpolation method, the Nelson–Siegel–Svensson method and the B-spline methods, used to determine zero coupon term structure from a few coupon bonds.

Section 5 is devoted to numerical implementations and comparisons. There we first consider a collection of American coupon bonds, and apply both the numerical procedures described in Sections 3 and 4 to them, and we run several consistency tests, in which we remove a coupon bond from the list, use the remaining ones to determine a zero coupon term structure, and then use the prices so determined to value the removed bond and compare its price to the original price and compute the relative deviation error.

When considering the German bonds, the non entropy based methods considered in Section 4 fail to converge or produce an inadequate zero coupon bond structure. Also, the entropy based methods fail to converge properly. As a possible