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Q1 # A cellular automata traffic flow model considering the heterogeneity of acceleration and delay probability

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HIGHLIGHTS

- The heterogeneity in the acceleration of vehicles is considered.
- The heterogeneity of the delay probability of vehicles is induced.
- There are two types of bistable phases.
- Three typical phases in the model are identified.
- The low and high velocity vehicles play an important role in synchronized flow.

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ABSTRACT

This study examines the cellular automata traffic flow model, which considers the heterogeneity of vehicle acceleration and the delay probability of vehicles. Computer simulations are used to identify three typical phases in the model: free-flow, synchronized flow, and wide moving traffic jam. In the synchronized flow region of the fundamental diagram, the low and high velocity vehicles compete with each other and play an important role in the evolution of the system. The analysis shows that there are two types of bistable phases. However, in the original Nagel and Schreckenberg cellular automata traffic model, there are only two kinds of traffic conditions, namely, free-flow and traffic jams. The synchronized flow phase and bistable phase have not been found.

1. Introduction

Traffic science examines the fundamental laws and properties of transportation systems. The traffic dynamic behavior of vehicular traffic is complex and has diverse, interesting, non-equilibrium features such as collective behavior, self-organization, coexisting phases, etc. [1–7].

Freeway traffic flow is a very complex spatiotemporal nonlinear dynamic process [1]. For this reason, empirical traffic pattern features are not fully understood. The three-phase traffic flow theory can explain empirical spatiotemporal traffic patterns better than earlier traffic flow theories [8–10]. Previous experimental studies have shown that the complexity in traffic flow is linked to diverse space–time transitions between three basically different kinds of traffic: free traffic flow,

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Table 1
Delay probability depends on the velocities of the vehicles.

$v(t) = gap$	1	2	3	4	5	···	*Linear function expression*
$p(v)$	0	0.1	0.2	0.3	0.4	···	$p(v) = [v(t) - 1]/2v_{max}$

synchronized traffic flow, and traffic jams [11]. Many existing cellular automata traffic flow models are based on Kerner's three-phase traffic flow theory [12–29]. The cellular automata (CA) model is useful for simulating large systems and a variety of CA models have been proposed [3,30–33,29,34–37]. Nagel and Schreckenberg (NS) [36] proposed a model of gradually increasing velocity, in which a vehicle may increase by only one unit per time step. In the Fukui–Ishibashi (FI) model [37], vehicles can move only m sites in one time step, and only if they are not blocked by vehicles in front. The notation m, which denotes the maximum velocity of vehicles, must be a positive integer. It is evident that these two popular one-dimensional traffic flow models are based on different vehicle accelerations. In real traffic, vehicle acceleration depends on their own current velocity, vehicle performance, driving habits, and so on; therefore, real acceleration must be a random number somewhere between 0 and the maximum velocity of vehicles.

The randomization of update rules of the NS and FI models [36,37] can provide the key to modeling the formation of spontaneously emerging traffic jams and natural speed fluctuations caused by human behavior or varying external conditions. In the NS model, if the velocities of vehicles are greater than zero, all vehicles have the same delay probability [36]. The FI model [37] has similar results if the velocities are equal to 5. However, Brilon and Wu argued that this rule has no theoretical basis and is in fact heuristic [38]. Randomization occurs inevitably in actual traffic and it is evident that different drivers have different delay probabilities under various velocities, external conditions, etc.

The heterogeneity of vehicular traffic is an important feature in traffic flow studies. Different types of driver's individual property and/or vehicles (e.g., vehicles with different maximal velocity, length, and so on) have been considered in some studies [39–53]. Unlike these studies, this study establishes a new cellular automata traffic flow model that considers the heterogeneity in the acceleration and delay probability of vehicles. Our model can reproduce some common characteristics of the real traffic, such as the start-and-stop waves, and present the synchronized flow phase and bistable phase. The fundamental diagram of the model shows quantitative coincidence of maximum flow with values taken from real traffic measurements.

2. Definition of the CA traffic model

For the sake of completeness, we now briefly define the NS model. In the NS model, a road is composed of L cells of equal size, and every single cell can be empty or occupied by a single vehicle. The velocity of each vehicle can be one of the $v_{max}+1$ allowed integer values, i.e., $v = 0, 1, \ldots, v_{max}$. Here, v_{max} (the notation v_{max} is a positive integer) denotes the maximum velocities of vehicles. Let v_{max} be 5 in this study. Below, the periodic boundary conditions are adopted and no vehicle is allowed to overtake on the road.

As cellular automata are dynamic systems that are discrete in nature, the acceleration of each vehicle must be a random integer. However, the acceleration values of vehicles can be no larger than their maximum velocities. Thus, the acceleration of each vehicle is also one of the $v_{max} + 1$ allowed integer values, $v = 0, 1, \ldots, v_{max}$.

As mentioned above, we only consider the effect of velocity on the delay probability of vehicles. The delay probability of each vehicle exists when the value of its possible velocity equals the value of its corresponding gap, and the probability varies with the size of the corresponding gap. We assume that the delay probability of each vehicle increases linearly with its velocity. As Table 1 shows, $p(v) = 0$ where $v(t) = 1$. In other words, the vehicle does not need to slow down if $v(t) = gap = 1$, which is similar to the conditions in Ref. [30].

The state of the road at time $t + 1$ can be obtained from time step t by simultaneously applying the following rules to all vehicles (parallel dynamics): (i) the randomization parameter of acceleration a is determined by $a = rand()/(v_{max} + 1)$, where $rand()$ function is a random function that can generate random numbers. (ii) Acceleration, velocity is $v_i(t + 1) \to \min(v_i(t) + a, V_{max})$; (iii) deceleration due to other vehicles, velocity is $v_i(t + 1) \to \min(v_i(t + 1), gap)$, where gap is the number of empty cells in front of the ith vehicle; (iv) the parameter of the delay probability is determined by $p = p(v)$; (v) randomization with probability p if $v_i(t + 1) = gap$, $v_i(t + 1) \to \max(v_i(t + 1) - 1, 0)$; and (vi) vehicle movement is $x_i(t + 1) \to x_i(t) + v_i(t + 1)$, where $v_i(t)$ and $x_i(t)$ are the velocity and the position of the ith vehicle at the current t time step, respectively.

In computer simulations, the length of each cell is set to 7.5 m. System size L is assumed to be 2000, which corresponds to an actual road length of around 15 km. One time step approximately corresponds to 1 s in real time. Thus, the maximum velocity $v_{max} = 5$ corresponds to 135 km/h in real traffic.

3. Rich behavior of the present model

The fundamental diagrams (FD) of the present model and the NS model are presented in Fig. 1(a). There are only two kinds of traffic conditions, namely, free-flow and traffic jam in the NS model [33]. However, in the present model, there are three phases: (i) free-flow, (ii) synchronized flow, and (iii) wide moving jam. Moreover, from Fig. 1(a), the maximum flow