Physica A xx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Q1 Weak dissipative effects on trajectories from the edge of basins of attraction

Q2 C.A. Jousseph [a,b], T.S. Kruger [b], C. Manchein [c], S.R. Lopes [b], M.W. Beims [b,*]

[a] Departamento de Física, Universidade Tecnológica Federal do Paraná, 85884-000 Medianeira, Brazil
[b] Departamento de Física, Universidade Federal do Paraná, 81531-980 Curitiba, Brazil
[c] Departamento de Física, Universidade do Estado de Santa Catarina, 89219-710 Joinville, Brazil

HIGHLIGHTS

- Convergency properties of trajectories under small dissipation.
- Dissipative holes in conservative tori.
- Lyapunov vectors in weakly dissipative systems.

ARTICLE INFO

Article history:
Received 15 March 2016
Available online xxxx

Keywords:
Dissipation
Standard map
Hyperbolicity

ABSTRACT

The purpose of this work is to present convergency properties of regular and chaotic conservative trajectories under small dissipation. It is known that when subjected to dissipation, stable periodic points become sinks attracting the surrounding trajectories which belong to rational/irrational tori, while chaotic trajectories converge to a chaotic attractor, if it exists. Using the standard map and a mixed plot we show that this simple scenario can be rather complicated and strongly depends on the dissipation intensity. For small dissipations the huge amount of attractors of the phase-space generates a complicated and intricate dynamics where trajectories are steered to their attractors based on the local (non)hyperbolicity, measured by the Lyapunov vectors. Dissipation creates holes (or attracting channels) on the torus from the conservative limit and allow trajectories to penetrate it. These holes are regions of large local hyperbolicity and are related to sticky channels reported recently. For stronger dissipation sinks tend to attract all trajectories, prevailing over the chaotic attractor.

Q3 Q4 Q5

1. Introduction

A suitable understanding of the behavior of trajectories in a dynamical system is essential for many applications including heat and confinement of particle in fusion plasmas devices, astrophysics, fluid mixing, particle accelerators, and chemical reactions [1]. Typical conservative systems present a mixed dynamics which consists in the coexistence of regular and chaotic motions. For two dimensional systems these motions are clearly separated by KAM (Kolmogorov, Arnold and Moser) [2] curves which prohibit chaotic orbits to penetrate the regular torus or even to reach all points in a constant energy surface. Such prohibition also prevents the unbounded increase of energy in the Fermi acceleration [2,3]. Nevertheless, the interface between phase-space areas related to both motions is not a smooth surface and the dynamics near the edge between chaotic

* Corresponding author.
E-mail address: mbeims@fisica.ufpr.br (M.W. Beims).

http://dx.doi.org/10.1016/j.physa.2016.03.015
0378-4371/© 2016 Published by Elsevier B.V.

and regular regions is very complex, leading to generic auto-similar structures [4]. One of the main effects of such interfaces is the presence of stickiness [1], which plays a crucial role in understanding distinct physical contexts, namely in anomalous transport and superdiffusion [5], non-equilibrium systems [6], the characterization of the dynamics in high-dimensional systems [7], transport in plasma devices [2], quantum optics [8], among many others [9–13].

In this work we connect the regular and chaotic dynamics by including a small amount of dissipation via a damping constant which multiplies the momentum. There are obviously other ways (rather more complicated) to introduce dissipative effects in a system of interest [14–23]. By adding small dissipation in Hamiltonian systems, stable periodic points become sinks attracting many surroundings rational/irrational curves and all KAM curves are destroyed [24,25]. The basin of attraction is obtained numerically and stable and unstable manifolds pave the path that trajectories converge to the sinks. A transient chaotic motion exists whose lifetime, after which the trajectory is attracted to an island sink, can be determined [24] and exhibits features of bifurcations [26]. The huge amount of attractors in the weak dissipative limit [27–29] makes the dynamics very rich and complex [30] since the persistent chaotic motion is densely interwoven with the regular one. Consequently it is very hard to tell *a priori* to which attractor trajectories from the edge of two (or more) basin of attractions will converge. In other words, what makes such trajectories from the edge converge to a given stable/unstable attractor and what defines the nature of the basin of attraction? The present work is an attempt to understand related question.

Here the conservative to small dissipative transition is studied for the dissipative standard map [31] in two ways. Firstly we analyze the changes which occur along one line of the basin of attraction as dissipation increases. This reveals the complex and fractal nature of the basin of attraction. Basin size evolutions as dissipation changes have been studied in the Hénon map [32], in autonomous and driven maps as the dissipation is increased [33] and attractors are mainly generated (annihilated) from elliptic (hyperbolic) periodic orbits from the conservative limit [26]. Secondly, we use Lyapunov vectors (LVs) which became recently a powerful tool to get new insights about the dynamics in nonlinear systems [34–39]. Using the LVs it is possible to determine the angles between the stable and unstable manifolds for each point in the phase-space. This allows us to identify *hyperbolic holes* along the KAM tori, where the chaotic trajectory escapes/penetrates more easily when dissipation is added. The holes act like channels which allow chaotic trajectories to reach the stable attractor. The role of such hyperbolic regions was used to describe sticky channels in the conservative limit [40], where chaotic orbits escape from the sticky motion via the hyperbolic regions.

The paper is organized as follows, Sections 2 and 3 introduce respectively the dissipative standard map model and the phase-space dynamics for parameters which are relevant for the analysis performed in this work. Section 4 presents our main results, which are the evolution of a line of attraction as the dissipation increases and the LVs analysis showing the holes in the torus where trajectories escape or are attracted. In Section 5 we summarize our results.

2. The model

It is appropriate to present results using a well known general model with wide applications, the dissipative Chirikov–Taylor standard mapping, which is given by [41]

$$\begin{cases} p_{n+1} = \epsilon\, p_n + (K/2\pi)\sin(2\pi x_n) \mod 1, \\ x_{n+1} = x_n + p_{n+1} \mod 1. \end{cases} \tag{1}$$

Here K is the nonlinearity parameter, x_n, p_n are respectively position and momentum at discrete times $n = 1, 2, \dots, N$, and $\epsilon = 1 - \gamma$ is the dissipation parameter defined in the interval $0 \le \gamma \le 1$. For our purpose it is worth to mention that for the conservative case $\epsilon = 1$, period-1 (shortly written per-1) fixed points are $p_1 = 1/2m$ (m integer) and $x_1 = 0, \pm 1/2$. The point $x_1 = 0$ is always unstable while $x_1 = \pm 1/2$ becomes unstable for $K > 4$. For higher periods there are primary families of periodic points (which exist in the limit $K \to 0$) and bifurcation families which are born only for larger values of K (for more details about the dynamics of the conservative and dissipative cases we refer the readers to Refs. [2,4,24]).

3. The phase-space dynamics

We start showing the general known behavior of stable and unstable periodic orbits when dissipation is introduced. Fig. 1(a) shows the dynamics for the conservative case ($\epsilon = 1$). The per-1 stable point is observed and four stable elliptic points. Since these stable points will be discussed in more detail later, we use the following notation along the line $p = 0$: $P_1^{\pm}$ located in phase-space at $(\pm 0.50, 0)$ and $P_4^{\pm}$ located at $(\pm 0.32, 0)$. The sign refers to the sign of x, or the location of the stable points, *i.e.* left for negative values of x and right for positive. These stable points are surrounded by their corresponding KAM rational/irrational tori and also by some smaller elliptic points not visible in the resolution from Fig. 1(a). The remaining Q6 phase-space dynamics is chaotic, beside some tiny stable points, also not visible. By adding a small amount of dissipation ($\gamma = 10^{-3} \to \epsilon = 0.999$) to the dynamics, we observe in Fig. 1(b) that rational/irrational tori lose their energy and roughly converge to the stable elliptic point their belong to. This is a very typical and well known behavior in the conservative to dissipative transition. While elliptic points are transformed into sinks and surrounding orbits converge to it, the chaotic motion becomes transient and may converge to a chaotic attractor (if it exists). In fact, even though this behavior seems to be simple, it is not. We have to remember that this is the origin of the basin of attraction in dissipative systems, well known to