Physica A 457 (2016) 377–390

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Game of collusions

Krzysztof Malarz *, Krzysztof Kułakowski

AGH University of Science and Technology, Faculty of Physics and Applied Computer Science, al. Mickiewicza 30, 30-059 Krakow, Poland

ARTICLE INFO

Article history:
Received 11 January 2016
Available online 6 April 2016

Keywords:
Social and economic systems
Social organizations
Anthropology
Complex systems
Structures and organization in complex systems

ABSTRACT

A new model of collusions in an organization is proposed. Each actor $a_{i=1,\dots,N}$ disposes one unique good $g_{i=1,\dots,N}$. Each actor a_i has also a list of other goods which he/she needs, in order from desired most to those desired less. Finally, each actor a_i has also a list of other actors, initially ordered at random. The order in the last list means the order of the access of the actors to the good g_i. A pair after a pair of actors tries to make a transaction. This transaction is possible if each of two actors can be shifted upwards in the list of actors possessed by the partner. Our numerical results indicate, that the average time of evolution scales with the number N of actors approximately as $N^{2.9}$. For each actor, we calculate the Kendall's rank correlation between the order of desired goods and actor's place at the lists of the good's possessors. We also calculate individual utility functions η_i, where goods are weighted according to how strongly they are desired by an actor a_i, and how easily they can be accessed by a_i. Although the individual utility functions can increase or decrease in the time course, its value averaged over actors and independent simulations does increase in time. This means that the system of collusions is profitable for the members of the organization.

1. Introduction

An ambitious and careful program of social research on organizations, known as Strategic Analysis (SA), has been framed nearly 40 years ago by Crozier and Friedberg [1]. The key idea of SA is that the structure and activity in organizations are shaped by the relations of power. Further, an actor is autonomous, hence her/his behaviour is uncertain and this uncertainty is an ingredient part of the actor's power over other actors. Further, each organization is contingent; it comes out as an artificial construct to solve a given problem. Therefore any theory which tries to derive a characteristics of an organization from 'natural' conditions is doomed to failure. These frames make SA applicable to qualitative discussions of specific case studies as of Airbus [2], Lidl [3], or selected (unnamed) universities [4], but particularly difficult to be translated into terms of a formal model of some generality. Also, any such translation contains elements which are necessarily arbitrary.

Yet, only recently the task has been undertaken in a few directions. Particular configurations of the relations of power in an organization (principal, supervisor, actor) have been investigated by Vafaï [5–7], who analysed couplings between different kinds of collusions in terms of mathematical theorems. The approach by Sibertin-Blanc and coworkers [8–10] develops the frames of SA in a consistent and methodical way. In this approach, each actor controls some relations which other actors can profit by. Also, each actor distributes his own stakes over the relations he/she can profit by. As an outcome of this game, played by all actors simultaneously, they get their capacities, which depend on both the relations and the stakes. In Ref. [10], the algorithm – termed as SocLab – is applied to the case of management of floods of the Touch river in France.

* Corresponding author.
E-mail addresses: malarz@agh.edu.pl (K. Malarz), kulakowski@fis.agh.edu.pl (K. Kułakowski).
URL: http://home.agh.edu.pl/malarz/ (K. Malarz).

http://dx.doi.org/10.1016/j.physa.2016.03.068
0378-4371/

The aim here is to analyse in detail one particular aspect of SA, i.e. binary collusions. By a collusion we mean, after Ref. [11], *secret agreement or cooperation especially for an illegal or deceitful purpose*. We are not interested, however, in specifying to what extent the collusions to be simulated are illegal or even deceitful. We are going to concentrate on the process of establishing a hierarchy in an organization. Similarly to the papers reported above, our motivation is to refer to the frames of SA, as described by Crozier and Friedberg [1]. Yet, it seems to us that the process of setting of hierarchies (ST) is at least as close to SA as the algorithm SocLab [10]. In particular:

- as stated in Ref. [1], there are always some relations of power in an organization; a simulation of ST should reproduce their dynamics;
- as stated in Ref. [1], an organization itself is a social construct; so is the hierarchy, constructed individually by each actor;
- the actors are of limited rationality and limited information; therefore they do not plan their behaviour in long time scale, just – as also stated in Ref. [1] – catch occasions;
- for the same reason, actors can neither predict nor control the behaviours of other actors, as those maintain their spheres of uncertainty. A successful cooperation with one actor does not preclude its future breaking for the sake of a more profitable cooperation with another actor;
- a game composed of binary subgames seems more realistic than a collective reorientation of relations, with perfect information available immediately for all actors.

Last but not least, as it will be shown below, the only parameter of our model is the number of actors. Obviously, our approach is rather explanatory than predictive. Yet it seems to us worthwhile to try to understand a selected process of SA, even if its description remains separated from particular case studies.

Our scenario is as follows. In a set of actors, each actor disposes some kind of resources. Also, each actor needs resources of all other kinds, with his/her individual order of needs. Finally, each actor has a list of all other actors, with their hierarchy equivalent to their order in the list. Once two actors simultaneously find that it would be profitable to be advanced in the hierarchy of the other actor, they shift each other upwards by one position. Then, the lists of hierarchies are the only elements which evolve.

In Section 2, our algorithm is explained in detail. Section 3 presents results of computer simulations. Finally, Section 4 contains discussion of the results and conclusions.

2. Model

Let $\mathbf{A} = \{a_1, \dots, a_N\}$ and $\mathbf{G} = \{g_1, \dots, g_N\}$ denote sets of N actors and N goods, respectively. Every actor i possess single and unique good $g_i \in \mathbf{G}$. Every actor i desires all other $(N-1)$ goods in a given order

$$\vec{\xi}_i = \mathcal{P}(\mathbf{G}_i), \tag{1}$$

where $\mathcal{P}(\mathbf{G}_i)$ is a random permutation of set $\mathbf{G}_i = \mathbf{G} \setminus \{g_i\}$. A sequence $\vec{\xi}_i$ do not change during simulation. Every actor i orders sequentially all other $(N-1)$ actors. This order is represented as a sequence $\vec{\zeta}_i^t$ which may evolve during time t. Initially, the list $\vec{\zeta}_i^0$ is chosen randomly, i.e.

$$\vec{\zeta}_i^0 = \mathcal{P}(\mathbf{A}_i), \tag{2}$$

where $\mathcal{P}(\mathbf{A}_i)$ stands for a random permutation of set $\mathbf{A}_i = \mathbf{A} \setminus \{a_i\}$.

In each time step t a pair (a_m, a_n) of actors is selected randomly. Let k_m denotes position of actor a_m in the list $\vec{\zeta}_n^{t-1}$ and k_n denotes position of actor a_n in the list $\vec{\zeta}_m^{t-1}$. If simultaneously, item (k_m-1) on the list $\vec{\zeta}_n^{t-1}$ and item (k_n-1) on the list $\vec{\zeta}_m^{t-1}$ represent actors which have goods considered by actors a_m and a_n as less valuable than goods of actors in positions k_m and k_n, then *transaction* takes place. During transaction actors in positions (k_m-1) and k_m on the list $\vec{\zeta}_n^t$ are swapped. The same procedure is realized on the $\vec{\zeta}_m^t$ list, i.e. actors in positions (k_n-1) and k_n are swapped as well. We call promoted and demoted actors 'winners' and 'losers' of the transactions, respectively.

The results are averaged over M independent simulations, i.e. various sequences of $(g_1, g_2, \dots, g_N)$, various hierarchies of actors goods $\vec{\xi}_i$ $(i = 1, \dots, N)$ (Eq. (1)) and various initial sequences $\vec{\zeta}_i^0$ (Eq. (2)).

2.1. An example of model rules application

To illustrate the model's rules let us consider a group of $N = 6$ actors with their goods g_i, hierarchy of goods $\vec{\xi}_i$ and initial hierarchy of actors $\vec{\zeta}_i^0$: