Physica A 457 (2016) 598–606

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Enhancing quantum Fisher information by utilizing uncollapsing measurements

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HIGHLIGHTS

- We propose an effective scheme for enhancing quantum Fisher information.
- The optimal measurement strength is only related to the weight parameter.
- The scheme can be extended to N-qubit pure state and open system.

ARTICLE INFO

Article history:
Received 13 October 2015
Received in revised form 30 March 2016
Available online 11 April 2016

Keywords:
Quantum Fisher information
Parameter estimation
Quantum uncollapsing measurements

ABSTRACT

As an indicator of estimation precision, quantum Fisher information (QFI) lies at the heart of quantum metrology theory. In this work, an effective scheme for enhancing QFI is proposed by utilizing quantum uncollapsing measurements. Two kinds of strategies for the arbitrary two-qubit pure state with weight parameter and phase parameter are implemented under different situations, respectively. We derive the explicit conditions for the optimal measurement strengths, and verify that the QFI can be improved quite well. Meanwhile, due to the relation of quantum correlation and QFI, the maximal value of QFI associated with phase parameter for pure state is always equal to 1. It is worth noting that the optimal measurement strength is only related to the weight parameter, as uncollapsing measurements operation does not induce any disturbance on the value of phase parameter. The scheme also can be extended to improve the parameter estimation precision for an N-qubit pure state. In addition, as an example, the situation of an arbitrary single-qubit state under amplitude damping channel is investigated. It is shown that our scheme also works well for enhancing QFI under decoherence.

1. Introduction

As one of the most fundamental missions in information theory, parameter estimation has been generalized to quantum field [1–3] due to the rapid development of quantum information theory and quantum technology. Fisher information, originally introduced by Fisher [4], plays a centric role in both statistical theory and quantum metrology. It quantifies the information that one can draw an associated parameter from a probability distribution. Generally speaking, the larger the value of Fisher information, the higher parameter precision we can estimate. Quite recently, quantum Fisher information (QFI), which is the extension of Fisher information in quantum regime, has attracted more and more attention [5–16]. Since QFI can capture the immanent sensitivity of the given system in regard to the change of certain specific parameters, it

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http://dx.doi.org/10.1016/j.physa.2016.04.001
0378-4371/

generally unveils more intricate performance than other correlation measures such as entanglement and quantum discord. Not only is it an available tool to evaluate the accuracy limits of quantum measurements but also it can accomplish various quantum information tasks in entanglement detect [7], investigation of uncertainty relations [17] and so on. Furthermore, as a crucial measure of information content for quantum state, QFI can describe the sensitivity of the state about disturbance of the parameter. Hence, how to enhance the precision of parameter estimation is the primary topic in quantum metrology, meanwhile it is of significant applications in quantum technology such as measurement of gravity accelerations [18], quantum frequency standards [19], clock synchronization [20], etc.

On the other hand, considerable attention [21–27] has been concentrated on uncollapsing measurement, since it can protect entanglement from decoherence and have been experimentally realized in solid system [28], linear optic devices [29,30], and superconducting phase qubits [31,32]. It also has been found to be useful in entanglement amplification [33]. Essentially, the so-called uncollapsing measurement can be seen as generalizations of Von Neumann measurements and associated with a positive-operator valued measure (POVM). The corresponding maps of quantum uncollapsing measurement with strength m and measurement reversal with strength r on one qubit in the computational basis $\{|0\rangle, |1\rangle\}$ can be described as a non-unitary quantum operation $M = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-m} \end{pmatrix}$ and $R = \begin{pmatrix} \sqrt{1-r} & 0 \\ 0 & 1 \end{pmatrix}$, respectively.

In this article, we focus our attention on probing an efficient way to improve QFI via uncollapsing measurements. Here the parameterized two-qubit pure state is considered and two strategies can be effectively implemented for different initial state. The merit of our scheme is that the uncollapsed state caused by pre-uncollapsing measurements can be certainly revived to the initial case in probability via suitable post-reversal measurements. It is worth mentioning that the scheme can be expanded to the case of an N-qubit pure state and open system. This paper is organized as follows. In Section 2, QFI is reviewed briefly. In Section 3, we propose a method for enhancing QFI via uncollapsing measurements in detail. In Section 4, we give a simple example to show our scheme also works well for open system. Finally, a summary is given in Section 5.

2. Quantum Fisher information

We briefly review parameter estimation theory and provide the analytical computational method of QFI in this subsection. Assuming an N-dimensional quantum state ρ_η which depends on an unknown parameter η, one could implement a set of quantum measurements $\{M(\varsigma)\}$ to extract information about η from ρ_η. According to the classical estimation theory, the quality of any measurement can be evaluated by Fisher information [1,2],

$$f'_\eta = \int \mathrm{d}\varsigma\, p(\varsigma|\eta) \left[\frac{\partial \ln p(\varsigma|\eta)}{\partial \eta}\right]^2 \tag{1}$$

where $p(\varsigma|\eta) = Tr[M(\varsigma) f_\eta]$ is the conditional probability of acquiring the measurement result ς when the value of the parameter is η.

By optimizing over all possible measurements, QFI [34] can be defined as

$$f_\eta = \max_{M(\varsigma)} f'_\eta = Tr(\rho_\eta L_\eta^2), \tag{2}$$

where L_η is the so-called symmetric logarithmic derivative (SLD) and satisfies

$$\frac{\partial \rho_\eta}{\partial \eta} = \frac{1}{2}(\rho_\eta L_\eta + L_\eta \rho_\eta), \tag{3}$$

here, the complete set of eigenvectors of L_η composes the optimal POVM to reach the QFI. Obviously, the calculation of QFI can be an easy task if the explicit form of the Hermitian operator L_η is known. However, unfortunately, only for several special cases, the analytical solution of L_η is simply found by utilizing some mathematical tricks [35–38].

From the perspective of geometry, owing to the spontaneous relation between QFI and Bures distance, the expression of QFI also can be directly acquired by utilizing the following formula

$$\mathrm{D}_B^2(\rho_\eta, \rho_{\eta+\mathrm{d}\eta}) = \frac{1}{4} f_\eta \mathrm{d}\eta^2, \tag{4}$$

where the Bures distance can be read as

$$\mathrm{D}_B(\rho, \sigma) = \sqrt{2}(1 - Tr\sqrt{\rho^{1/2}\sigma\rho^{1/2}})^{1/2}. \tag{5}$$

That is, if we can get the explicit form of Bures distance of the corresponding states, then the formula of QFI can be obtained straightforwardly by utilizing this relation. Although this expression has already been exploited in several situations [39,40], nevertheless, it cannot be ignored that this strategy is somewhat difficult to use especially in multipartite system.

On the other hand, by employing the spectral decomposition $\rho_\eta = \sum_{i=1}^K q_i |\phi_i\rangle\langle\phi_i|$ (q_i and $|\phi_i\rangle$ are the eigenvalue and eigenvector of ρ_η), in terms of the subset $\{|\phi_i\rangle\}$ with $q_i \neq 0$, QFI has a more expedient way that can be expressed as [41,8]

$$f_\eta = f_C + f_Q \tag{6}$$