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Bosonic binary mixtures with Josephson-type interactions

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HIGHLIGHTS

- A bosonic binary mixture with imbalanced interactions is studied.
- An effective one-loop potential for an $O(2)$ model is computed.
- Condensate fractions as functions of temperature and chemical potential are presented.

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ABSTRACT

Motivated by experiments in bosonic mixtures composed of a single element in two different hyperfine states, we study bosonic binary mixtures in the presence of Josephson interactions between species. We focus on a particular model with $O(2)$ isospin symmetry, lifted by an imbalanced population parametrized by a Rabi frequency, Ω_R, and a detuning, ν, which couples the phases of both species. We have studied the model at mean-field approximation plus Gaussian fluctuations. We have found that both species simultaneously condensate below a critical temperature T_c and the relative phases are locked by the applied laser phase, α. Moreover, the condensate fractions are strongly dependent on the ratio $\Omega_R/ \mid \nu \mid$ that is not affected by thermal fluctuations.

1. Introduction

Multicomponent quantum gases are fascinating systems [1]. Basic research in this area has enormously grown in the last few years [2]. Due to the ability of optically trapping and cooling gases to extremely low temperatures, it is possible to study different phenomena in bosonic [3,4] as well as fermionic mixtures [5]. Important quantum effects like Bose–Einstein condensation (BEC) and superconductivity can now be studied in a very controlled way in multicomponent atomic systems.

Interesting experiments with mixed bosonic quantum fluids have been done by simultaneously trapping ^{87}Rb atoms in two different hyperfine states [6–9]. The relative population is reached by applying a coupling field characterized by a Rabi frequency Ω_R and a detuning ν with respect to the spacing between the energy levels of the two hyperfine states. In this way, it is possible to transfer atoms from one hyperfine state to the other, producing a Josephson-type interaction between species [10–12].

In general, the name "Josephson interaction" refers to the interaction of a large number of bosonic degrees of freedom allowed to occupy two different quantum states. Although it was originally proposed in superconductor systems [13], where

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the bosons are Cooper pairs, there are many other systems where this effect shows up. A review covering different physical systems can be found in Ref. [14]. We can distinguish two types of Josephson effects [15]: the so-called "external", where the two states are spatially separated, like, for instance, in BEC trapped in a double-well potential [16–19], or the "internal", where the two bosonic states are interpenetrated, without geometrical distinction, like, for instance, the experiments in Refs. [7,8]. In this paper, we are mainly interested in the latter case of internal Josephson-type interactions.

Static and dynamical properties of binary bosonic mixtures in different trap geometries have been studied theoretically by essentially using Gross–Pitaevskii equations [20–26]. Moreover, to study properties of uniform condensates, especially those issues related with fluctuations, such as symmetry restoration, reentrances, etc., quantum field theory at finite density and temperature [27–30] is a useful technique. Related models, such as $O(N)$ models, have also been extensively studied by using large-N approximation and renormalization-group techniques [31,32]. These papers are mostly concentrated in multicomponent systems which conserve the particle number of each species independently.

Motivated by these results, we decided to address the effect of Josephson-like interactions in uniform bosonic mixtures. For simplicity, we have considered an $O(2)$ model, perturbed with an explicit symmetry-breaking term parametrized by the Rabi frequency Ω_R and the detuning term ν. This model is analyzed in mean-field approximation plus Gaussian fluctuations.

In the absence of Josephson interactions, this model is at the onset of phase separation, since the two species are not physically distinguishable. However, the presence of Josephson interactions changes this scenario since it explicitly breaks $O(2)$ symmetry. There is a temperature regime where the two atomic species uniformly condensate at the same critical temperature T_c and their relative phase is locked by the phase of the applied electromagnetic field responsible for the Rabi coupling and the detuning. The relative population of each condensate strongly depends on the ratio $\Omega_R/|\nu|$. The main results of this paper are shown in Figs. 3 and 4 where we depict the condensate fraction of the two species as a function of temperature for different values of the parameter $\Omega_R/|\nu|$. Thus, controlling the external laser parameters, *i.e.*, the Rabi coupling, the laser frequency (essentially the detuning) and the phase, it is possible to control each one of the condensate fractions as well as its phase difference.

An important result is that, due to the original $O(2)$ symmetry, the effective Rabi frequency, given by $\Omega_{\text{eff}} = \sqrt{\Omega_R^2 + |\nu|^2}$ is strongly renormalized by thermal fluctuations. On the other hand, the ratio $\Omega_R/|\nu|$, that controls the bosonic mixture, remains unaffected by quantum as well as thermal fluctuations. Thus, the ratio between both condensates are temperature independent, allowing the possibility of control the relative condensate fractions with high accuracy.

The paper is organized as follows. In Section 2, we describe a general model for a binary mixture using quantum field theory language. In Section 3, we concentrate on the $O(2)$ model perturbed with Josephson interactions. In Section 4, we present the mean-field solution, while in Section 5 we analyze the effect of fluctuations. Numerical results are presented in Section 6 and, finally, we discuss our results in Section 7. We reserve a brief Appendix A to describe the definitions of Rabi frequency and detuning parameter used to built our model.

2. A quantum field theory for binary bosonic mixtures

We will consider two bosonic species described by two complex fields, $\phi(\vec{x}, t)$ and $\psi(\vec{x}, t)$. The model is defined by the action

$$S = \int d^3x dt \, \left\{ \mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_I \right\}, \tag{1}$$

where $\mathcal{L}_\psi$ and $\mathcal{L}_\phi$ are the non-relativistic quadratic Lagrangian densities

$$\mathcal{L}_\psi = \psi^* \left(i\partial_t + \frac{\nabla^2}{2m} + \mu_\psi \right) \psi, \tag{2}$$

$$\mathcal{L}_\phi = \phi^* \left(i\partial_t + \frac{\nabla^2}{2m} + \mu_\phi \right) \phi. \tag{3}$$

μ_ψ and μ_ϕ are the chemical potentials for the ψ and ϕ species, respectively. We choose the same mass m for both species, since we are interested in mixtures composed by a single element in two different hyperfine states.

It is convenient to parametrize the chemical potentials as

$$\mu_\phi = \mu + \Omega_R \tag{4}$$

$$\mu_\psi = \mu - \Omega_R. \tag{5}$$

The parameter μ controls the overall particle density at the time that the Rabi frequency Ω_R controls the population imbalance (see Appendix A for the microscopic physical meaning of Ω_R). Throughout the paper, we have used a unit system in which $\hbar = 1$.

The interaction Lagrangian density $\mathcal{L}_I$ can be split into two terms,

$$\mathcal{L}_I = \mathcal{L}_c + \mathcal{L}_J. \tag{6}$$