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Phase transitions in the majority-vote model with two types of noises

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HIGHLIGHTS

- Independent behavior as a second noise.
- Possibility to suppress the usual phase transition.
- Universality class of the equilibrium Ising model.
- Results on the fully-connected graph and on the square lattice.

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ABSTRACT

In this work we study the majority-vote model with the presence of two distinct noises. The first one is the usual noise q, that represents the probability that a given agent follows the minority opinion of his/her social contacts. On the other hand, we consider the independent behavior, such that an agent can choose his/her own opinion $+1$ or -1 with equal probability, independent of the group's norm. We study the impact of the presence of such two kinds of stochastic driving in the phase transitions of the model, considering the mean field and the square lattice cases. Our results suggest that the model undergoes a nonequilibrium order–disorder phase transition even in the absence of the noise q, due to the independent behavior, but this transition may be suppressed. In addition, for both topologies analyzed, we verified that the transition is in the same universality class of the equilibrium Ising model, i.e., the critical exponents are not affected by the presence of the second noise, associated with independence.

1. Introduction

The study of dynamics of opinion formation is nowadays a hot topic in the Statistical Physics of Complex Systems, with a considerable amount of papers published in the last years (see Refs. [1–4] and references therein). Even simple models can exhibit an interesting collective behavior that emerges from the microscopic interaction among individuals or agents in a given social network. Usually those models exhibit nonequilibrium phase transition and rich critical phenomena, which justifies the interest of physicists in the study of opinion dynamics [5–15].

The majority-vote model has been extensively studied in the last 20 years [5]. In the standard model, each site of a square lattice has an Ising-spin variable whose state ± 1 may be associated with the opinion of an individual in a given social community. The time evolution of the model is governed by an inflow dynamics, where the center spin is influenced by its nearest neighbors: an individual located at the central site adopts the minority sign of the spins in its neighborhood

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with probability q and the majority sign with probability $1 - q$. Numerical results indicate that the model undergoes a nonequilibrium phase transition at a critical noise $q_c \approx 0.075$, and the critical exponents were found to be the same as those of the equilibrium 2D Ising model, i.e., $\beta \approx 0.125$, $\gamma \approx 1.75$ and $\nu \approx 1.0$ [5]. The transition separates an ordered phase, where one of the opinions dominates the population, from a disordered one, where the two opinions coexist in equal proportions.

After that, many extensions of the original model were proposed. For example, we can highlight the interaction between two different classes of agents [6], spins with three-state variables [7,8], noise distributed according to a bimodal distribution [9], diffusion [10] and heterogeneous agents [12]. The influence of topology was also considered, with studies on random [13], hypercubic [14] and hyperbolic [15] lattices, random graphs [16], small-world [17,18] and scale-free networks [19,20], among others. Some new universality classes were found for different topologies [7,12–17], but in some of the above-mentioned modifications the critical exponents are Ising-like, i.e, they are the same as those of the original majority-vote model [6,8–10].

The consideration of the usual noise q in the majority-vote model produces an effect similar to the introduction of the *contrarians* in the population, individuals that adopt the choice opposite to the prevailing choice of the others, whatever this choice is [21–26]. Indeed, q is the probability of an agent assuming the opposite opinion shared by the local majority. The contrarian effect is called *anticonformism* in the language of Social Sciences, and it is one kind of nonconformism [27,28]. Another kind of nonconformism is *independence*, where the agent also take cognizance of the group norm, but he/she decides to take one of the possible opinions (± 1 in the case of the majority-vote model) independently of the majority or the minority opinion in the group [29,30]. Effects of independence on phase transitions in opinion models were considered recently [31–34].

In this work we consider the two mentioned kinds of nonconformity, anticonformity and independence. The last is introduced in the system as a probability p, in a way that a given agent chooses one of the two possible opinions with equal probability (1/2). Thus, the population evolves under the presence of two types of noises, represented by the parameters q and p. We consider agents placed on fully-connected networks and on square lattices, and our interest is to study the effects of the two kinds of stochastic driving in the phase transitions of the model.

This work is organized as follows. In Section 2 we present the microscopic rules that define the model, and the analytical and numerical results are discussed in two distinct subsections, considering the mean-field approximation and the square lattice. Finally, our conclusions are presented in Section 3.

2. Model and results

Our model is based on the majority-vote model [5]. Every site of a given lattice with N sites is occupied by an agent, and to each site i we assign a random opinion $\sigma_i = \pm 1$ with equal probability 1/2, corresponding to the two possible opinions in a certain subject. In the original model [5], a randomly chosen individual follows the opinion of the minority of its 4 nearest neighbors with probability q and adopts the majority sign of the spins in its neighborhood with probability $1 - q$. In this work we will consider the effects of the independent behavior, where a randomly chosen individual acts independently of their neighbors with probability p. In this case, he/she chooses one of the two possible opinions ± 1 with equal probability 1/2. On the other hand, with the complementary probability $1 - p$ we apply the original rule of the majority-vote model. Summarizing, considering the effects of the two noises q and p, a given spin σ_i is flipped with probability

$$w_i = \frac{1}{2}\,(1-p)\left[1 - \gamma\,\sigma_i\,S\left(\sum_{\delta}\sigma_{i+\delta}\right)\right] + \frac{p}{2}, \qquad (1)$$

where $\gamma = 1 - 2\,q$, $S(x) = \mathrm{sgn}(x)$ if $x \neq 0$, $S(0) = 0$ and the summation is over the nearest neighbors. Observe that in the absence of the independent behavior ($p = 0$) we recover the standard flip probability of the majority-vote model [5].

Thus, our model presents two distinct types of stochastic driving, governed by the two noises q and p. In the language of social sciences, we are considering two kinds of nonconformity, namely anticonformity (contrarian effect, parameter q) and independence (parameter p). We are interested in the critical behavior of the model. As the parameter p is the novelty of the model, we will consider the quantities of interest as functions of p, for typical values of q. These quantities are magnetization per spin, the susceptibility and the Binder cumulant, given by

$$m = \left\langle \frac{1}{N}\left|\sum_{i=1}^{N}\sigma_i\right|\right\rangle, \qquad (2)$$

$$\chi = N\,(\langle m^2\rangle - \langle m\rangle^2), \qquad (3)$$

$$U = 1 - \frac{\langle m^4\rangle}{3\,\langle m^2\rangle^2}, \qquad (4)$$

respectively. In Eqs. (2)–(4), $\langle \cdots \rangle$ denotes a configurational average taken at steady states.

In this work we will consider two distinct topologies for the model, the fully-connected network and the square lattice. These distinct cases will be treated separately in the following subsections.