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Application of quantum master equation for long-term prognosis of asset-prices

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HIGHLIGHTS

- The model is a contribution to the fields of econophysics and behavioral finance.
- Formalism of quantum physics is used to devise a dynamical model of asset price formation.
- The expectations of the agents on the finance market play a key role in this model.
- The notions of entanglement and superposition encode the non-classical uncertainty related to price formation.
- A numerical simulation of the model for two correlated assets is performed.

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ABSTRACT

This study combines the disciplines of behavioral finance and an extension of econophysics, namely the concepts and mathematical structure of quantum physics. We apply the formalism of quantum theory to model the dynamics of some correlated financial assets, where the proposed model can be potentially applied for developing a long-term prognosis of asset price formation. At the informational level, the asset price states interact with each other by the means of a "financial bath". The latter is composed of agents' expectations about the future developments of asset prices on the finance market, as well as financially important information from mass-media, society, and politicians. One of the essential behavioral factors leading to the quantum-like dynamics of asset prices is the irrationality of agents' expectations operating on the finance market. These expectations lead to a deeper type of uncertainty concerning the future price dynamics of the assets, than given by a classical probability theory, e.g., in the framework of the classical financial mathematics, which is based on the theory of stochastic processes. The quantum dimension of the uncertainty in price dynamics is expressed in the form of the price-states superposition and entanglement between the prices of the different financial assets. In our model, the resolution of this deep quantum uncertainty is mathematically captured with the aid of the quantum master equation (its quantum Markov approximation). We illustrate our model of preparation of a future asset price prognosis by a numerical simulation, involving two correlated assets. Their returns interact more intensively, than understood by a classical statistical correlation. The model predictions can be extended to more complex models to obtain price configuration for multiple assets and portfolios.

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1. Introduction

This paper is a contribution to the field of *behavioral finance*, with an emphasis on mathematical modeling of irrationality.[1]The problem that we search to approach in this study cannot be formulated better than in the following extended citation from Takahashi and Terano, [3]:

"Most conventional financial theories build the models by assuming representative rational investors based on the hypothesis that "even if each investor has different future prospects, these errors are canceled out as a whole and do not affect the prices". However, the participants of real markets only have limited information and they do not necessarily process the obtained information appropriately. There is a good possibility that the market participants have a common bias that deviates from rationality. If this is the case, the deviation from rationality will not be canceled out and will have a large impact on the prices. To unravel the mechanism of price fluctuation in real markets, it is quite important to understand the difference between the decision-making in the real world and the one based on the maximization of expected utility and to clarify how the prices are affected by the deviation from rationality on decision-making".

We proceed with modeling the formation of such biases[2] with the aid of the novel approach based on exploring the formalism of quantum theory (and its methodology) outside of physics. Recently, this formalism was actively applied to a wide range of social science problems, especially in cognitive psychology, behavioral finance and decision making, see Refs. [9–23]. At heart of this formalism is the quantum probability theory, based on the calculus of complex probability amplitudes (or more generally density matrices). This calculus leads to a nonclassical scheme of probability update, different from the classical Bayesian scheme. Thus, the quantum approach is able to capture violations of Bayesian rationality. For example, in models of behavioral economics it is important to explain and model contexts for which the *Savage Sure Thing Principle* can be violated. Effectively, in a situation where one cannot presume that agents are "Bayesian rational", a deeper type of uncertainty than the uncertainty of the rational agents, reasoning and acting in accord with some probabilistic information, can be considered. In the mathematical formalism of quantum theory such nonclassical uncertainty is represented in the form of *superposition of alternatives*. Please, see Remark 1 for a more extensive interpretation of the distinction, made between the classical and the quantum approaches to uncertainty.

Along with the other interdisciplinary applications, in this work, quantum physics is used as an operational formalism, describing statistical data generated by experiments. Its crucial difference from classical statistical theories is that it ignores all the parameters of the model, besides those directly related to the measurements.[3]

With this in mind, the superposition of state vectors, representing some choice alternatives, is just a mathematical tool to express uncertainty, which is deeper than the uncertainty described by a classical probability. The irrationality of decision making of the market agents that is manifest in the quantum superposition of their beliefs and decisions, is one of the main factors behind the deep quantum-like uncertainty in the formation of asset prices. Such a price formation of an asset is expressed in the form of superposition of the different prices and can yield a price that deviates from the fundamental value predicted by the classical finance theories. Another profoundly quantum information tool, which we use in our modeling is *entanglement*, i.e., the states' non-separability. The prices of a group of assets (e.g. in a portfolio) can be correlated with each other and with a "financial bath", by correlations that are stronger than the possible classical probabilistic correlations.

We devise a model that dynamically captures the process of asset price formation and can be applied for preparing a long-term financial prognosis. A decision maker can be e.g., an individual financial consultant, trader, financial forecast agency, or a financial trading corporation. In order to develop some future trading strategy the decision maker (we refer to her as a financial expert) would need to come with a prognosis for a group of asset prices. This model can be expanded for forecasting the movements of different types of financial instruments, e.g., currency exchange rates. We model the process of decision making related to the asset price formation. The resolution of uncertainty in future prices of assets is described with the aid of the theory of open quantum systems [26]: the most powerful and general apparatus to describe adaptive dynamics of a system (in our case assets) interacting with some bath (in this representation, the bath is of a purely informational nature, consisting of the expectations of the agents acting on the finance market, the mass-media news and other signals of financial, social, and political origins that can be regarded as "events" to contribute to price change).

[1] The concept of irrationality can be coupled to the notion "bounded rationality" coined by Herbert Simon [1,2] of finance market players. The deviations from the rational mode of processing the financial information and the implications it has for the formation of asset prices, will be discussed in Section 5.

[2] See also Shiller [4] who emphasized that financial markets are sometimes irrational as investors often make their decisions based on personal preferences and e.g. over-confidence bias. Irrationality of investors is actively explored in the studies related to the emergence of financial bubbles, e.g., Bailey [5], Kindleberger [6,7]. It is beyond the scope of this work to concentrate on the specific interrelation, irrationality-bubbles. We proceed under the assumption that the investors may behave irrationally even in the absence of bubble-like events, see agent based model simulation by Ref. [8]. Of course, we understand well the role of financial bubbles and the subsequent downturns in increasing the interest to behavioral finance. Bubbles call for a behavioral explanation. The recent financial crisis of 2008 clearly demonstrated that strict opponents of the Efficient Market Hypothesis, claiming that bubbles are impossible in well-organized markets, where all financial information is publicly available, have neglected some factors behind the real price formation of financial assets. Behavioral factors seem to be determining for the price evolution even on informationally-efficient markets, see, e.g., Bailey [5].

[3] In quantum physics, one even employs the so called no-go theorems for such parameters, for example, see Ref. [24]. However, for our purposes, these theorems are not of such a value as in physics. Consider, for example, one of the most famous no-go theorems, on Bell's inequality [25,24]. It states that if the "hidden parameters" were existing, they would be nonlocal. However, it seems that, e.g., for the financial markets, the issue of nonlocality is not critical. Definitely, financial correlations are nonlocal, but they are not super-luminary. Another no-go theorem is the Kochen–Specker theorem [24]. It says that if "hidden parameters" were existing, they would be contextual. Indeed, human behavior is intrinsically contextual and contextuality is a key feature of the quantum formalism, which is resonating with its notion in behavioral studies, cf. with Refs. [16,17].

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