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Stock market stability: Diffusion entropy analysis

Shouwei Li *, Yangyang Zhuang, Jianmin He

School of Economics and Management, Southeast University, Nanjing, 211189, China

HIGHLIGHTS

- We analyze the stock market stability based on normalized diffusion entropy.
- We analyze extreme cases of the stock market based on conditional entropy.
- The normalized diffusion entropy and conditional entropy can capture the financial crisis.

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ABSTRACT

In this article, we propose a method to analyze the stock market stability based on diffusion entropy, and conduct an empirical analysis of Dow Jones Industrial Average. Empirical results show that this method can reflect the volatility and extreme cases of the stock market.

1. Introduction

Following the recent financial crisis, understanding the stability of financial systems has attracted more extensive attention in different fields. Specially, the abnormal behaviors in financial markets have been analyzed in different methods, such as convexity, variance, VaR, ARCH and GARCH class models [1–7], and so on.

The abnormal behaviors in financial markets have also been analyzed using the concepts and methods developed from econophysics, where entropy is one of them. For example, McCauley [8] discusses whether entropy could reflect the uncertainty and disorder of stock markets. Bentes and Menezes [9] investigate the volatility of seven stock indexes based on Tsallis and the Shannon entropies. Zhou et al. [7] find that the information entropy method can be a better risk measurement of bonds than other methods, namely, convexity, variance and VaR. Oh et al. [10] analyze risk in the financial market in the return time series of several market indices based on entropy.

In this paper, we construct a method to measure the stability of stock markets based on diffusion entropy, where diffusion entropy is based on diffusion process and Shannon entropy [11]. Works closest to ours are those of Bentes and Menezes [9] and Oh et al. [10]. However, our paper is different from theirs. In this paper, we adopt diffusion entropy, while Bentes and Menezes [9] use Tsallis entropy and Shannon entropy, and Oh et al. [10] also use Shannon entropy. And our paper is different from Oh et al. [10] in the construction of the conditional entropy. The remainder of this paper is organized as follows. After this introduction, Section 2 describes the methodology used in analyzing the stock market stability. Section 3 presents the main results, and Section 4 provides a conclusion.

* Corresponding author.
E-mail address: lishouwei@seu.edu.cn (S. Li).

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2. Methodology

Let $\{\eta_i\}_{i=1}^{N}$ be a discrete financial time series, and N be the length of the series. The lth value of the time series can be treated as a particle located at position η_l from a one dimensional image. We adopt the diffusion algorithm of Scafetta and Grigolini [11] to calculate the diffusion entropy of $\{\eta_i\}_{i=1}^{N}$. For any given diffusion time t, $1 \le t \le N$, we can find $N - t + 1$ sub-sequences defined in the following:

$$\xi_i^{(s)} = \eta_{i+s} - \eta_{i+s-1}, \quad s = 0, 1, \ldots, N - t. \tag{1}$$

For any of these sub-sequences, we build up a diffusion trajectory by the position

$$X^{(s)}(t) = \sum_{i=1}^{t} \xi_i^{(s)}, \tag{2}$$

where $X^{(s)}(t)$ denotes the new position of the sth particle through the diffusion process.

In order to calculate the diffusion entropy, we divide the x axis into sections of length ε, where ε is assumed to be a fraction of the square root of the variance of the fluctuation η_i, and consequently independent of t. Let $N_i(t)$ denote the number of particles falling in the range of the ith section. Then we determine the probability that one particle may fall in such a section at time t, $p_i(t)$, by means of

$$p_i(t) = \frac{N_i(t)}{N - t + 1}. \tag{3}$$

At this stage, the diffusion entropy at time t is determined as follows:

$$E(t) = -\sum_i p_i(t) \ln[p_i(t)]. \tag{4}$$

Moreover, we define the normalized diffusion entropy as

$$H(t) = \frac{E(t)}{t}. \tag{5}$$

In order to distinguish extreme market status, such as a bear, or a bull market, similar to Oh et al. [10], we define conditional entropy as:

$$C(t) = 1 + \alpha[1 - H(t)], \tag{6}$$

where $\alpha = 1$, if $x > y$; $\alpha = -1$, if $x < y$; and $\alpha = 0$, if $x = y$. x and y denote the number of positive values and that of negative values for the series $\{\xi_i^{(t)}\}$ respectively. Therefore, α has a positive value for bull markets and a negative value for bear markets.

3. Empirical results

In this section, we investigate the stock market stability during the period of the recent financial crisis, using the entropy defined in the previous section. In this paper, we adopt the daily close price of Dow Jones Industrial Average (DJIA) from January 2006 to June 2010 to conduct the empirical analysis, which is shown in Fig. 1. We can see from it that the prices of the market during the financial crisis decrease, whereas they increase after the financial crisis. The total period is subdivided into three periods: the period from January 2006 to June 2007 as the period of the normal stock market states, the period from July 2007 to December 2008 as the period of the financial crisis, and the period from January 2009 to June 2010 as the period of the stock market recovery.

We now investigate the relationship between the entropy and the degree of the stock market stability. We find that the change of entropy values with the diffusion time $t = 5$ days coincides with the financial crisis, thus set the diffusion time $t = 5$ days in this paper. Fig. 2 presents the monthly normalized diffusion entropy values of DJIA from January 2006 to June 2010. In the first period, entropy values maintain at a relatively high level, which indicates the stability of the stock market. In fact, during the period from January 2006 to June 2007, the stock market price is at the growing stage and has less volatility. In the period of the financial crisis, the entropy value has significant fluctuations, and reduces to 0.211 in September 2008, which precisely reflects the sharp fall of the stock price in September. Compared with the period of the financial crisis, the stock market restores a certain vitality, and the change of entropy values is not very stable in the third period. This means that the stock market stability has suffered significantly from the financial crisis, and has not completely get out of it.

The result of Fig. 2 demonstrates that the diffusion entropy value performs well for the degree of the stock market stability. Oh et al. [10] also find that the estimation of stability of stock markets based on the entropy coincides with the financial crisis between 2007 and 2008. But, the difference of the method of this paper compared to that adopted by Oh