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Q1 Two dimensional velocity distribution in open channels using Renyi entropy

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HIGHLIGHTS

- Two dimensional velocity distribution in open channels is derived using Renyi entropy.
- Derived Renyi entropy-based velocity distribution agrees well with field as well as laboratory observations.
- An entropy parameter G is defined which simplifies the derived velocity distribution and characterizes open channel flow.

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ABSTRACT

In this study, the entropy concept is employed for describing the two-dimensional velocity distribution in an open channel. Using the principle of maximum entropy, the velocity distribution is derived by maximizing the Renyi entropy by assuming dimensionless velocity as a random variable. The derived velocity equation is capable of describing the variation of velocity along both the vertical and transverse directions with maximum velocity occurring on or below the water surface. The developed model of velocity distribution is tested with field and laboratory observations and is also compared with existing entropy-based velocity distributions. The present model has shown good agreement with the observed data and its prediction accuracy is comparable with the other existing models.

1. Introduction

Flow in an open channel can be laminar, turbulent or mixed type. But in natural situations, mostly turbulent flow occurs. In case of laminar flow, velocity can be defined accurately. But in turbulent flow, the velocity vector fluctuates both spatially and instantaneously and it is never stationary. Velocity distributions have been derived using either experimental or deterministic hydrodynamic methods. The time-averaged velocity component is traditionally treated to be deterministic without uncertainty. However, what is called an 'average' value can realistically be regarded only as a random variable and can be presented in terms of statistics as the mean and variance due to uncertainty in any sample averages. The true average may never be known. Entropy is a property of a system which measures the uncertainty or disorder within the system. In hydraulic studies, there are always uncertainties associated with the variables such as flow velocity, sediment concentration, and shear stress. Concept of entropy provides the foundation and connection between the deterministic and probabilistic world. Wide researches have been carried out since long to study velocity in turbulent flow through experimental or deterministic approach. In comparison to that, study on velocity through entropy theory is just a few and yet a lot to be explored. So the present work endeavors to study two dimensional velocity distribution in open channel using Renyi entropy.

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The variation of longitudinal velocity along transverse and vertical directions of an open channel is a long standing topic of research. In wide open channels, the velocity (longitudinal component) is assumed to increase monotonically in the vertical direction from the channel bed to the water surface and the maximum velocity occurs on the vertical line situated in the center of the channel cross section. However in natural open channels, the variation of velocity along both the vertical and transverse directions is important and hence it should be considered as two-dimensional. In the transverse direction the velocity is near zero at the boundaries and its maximum occurs somewhere in the middle of the channel but not necessarily in the center. In vertical direction, the velocity increases from zero at the channel bed to the maximum at or below the water surface near the channel center. The phenomenon that the maximum velocity occurs below the water surface is called the dip phenomenon. The dip phenomenon is an important feature of open channel flow and has created interest among researchers since long (Francis [1]; Stearns [2]).

Classical laws given by Prandtl–von Karman and the power law velocity distributions are all one dimensional. These velocity distributions are satisfactory for wide rectangular channels in which velocity varies in the vertical direction only. The entropy theory allows to construct an efficient method for describing velocity in both one and two dimensions. Introducing a new curvilinear coordinate system and employing Shannon entropy (Shannon [3]), Chiu [4] derived a 2D velocity distribution by assuming time averaged velocity as a random variable. He derived the location of maximum velocity also and a dimensionless parameter M_c which characterizes the velocity distribution. Later on, 2D formulation of Chiu [4] has been extended in a series of papers by Chiu and his associates. Chiu [5] presented a technique for estimating parameters of the 2D velocity distribution, Chiu [6] estimated energy and momentum coefficients in terms of the entropy parameter M_c. Chiu and Murray [7] described variation of velocity distribution along nonuniform open channel flow. They presented a graphical method to estimate velocity distribution parameters. Chiu and Said [8] established a relation between maximum and mean velocity in terms of the entropy parameter.

In a later period, another entropy which generalizes Shannon entropy, called Tsallis entropy [9] has been introduced for estimating 2D velocity distribution in open channel flow by Luo and Singh [10]. Similar to Chiu, they defined a dimensionless parameter M which describes the characteristics of open channel flows. To reduce the number of parameters in the coordinate system defined by Chiu [4], Marini et al. [11] developed a new method for describing 2D velocity distribution using Shannon entropy in which the cumulative distribution function (CDF) was hypothesized using x, y-coordinates. Cui and Singh [12] described 2D velocity distribution employing Tsallis entropy by using the CDF described by Marini et al. [11].

The literature shows that till now only Shannon and Tsallis entropies have been used for describing velocity distributions in an open channel flow. There is another entropy called Renyi entropy (Renyi [13]), which is the first formal generalization of Shannon entropy, has not been used for studying open channel flow. Renyi entropy has similar properties as the Shannon entropy namely it is additive and it has maximum entropy in case of equiprobability. On the other hand, both the Tsallis and the Renyi entropy contain an additional parameter α which can be used to make it more or less sensitive to the shape of the probability distribution. Maximum entropy principle gives exponential type least-biased probability distribution in case of Shannon entropy while it gives same kind of probability distributions in case of the Tsallis and the Renyi entropy as they are monotonous functions of each other. For deriving the velocity distributions in open channel using entropy theory, these entropies differ through the parameters they contain and the constraint equations. So it may be interesting to explore the use of Renyi entropy for modeling of velocity distribution. Therefore, the objective of this paper is to derive an entropy-based two dimensional velocity distribution model and to characterize the velocity distribution starting from the Renyi entropy. The derived velocity distribution is then tested by using field and experimental measurements and is also compared with Shannon and Tsallis entropy based velocity distributions.

2. Entropy theory for velocity distribution

The entropy theory to derive velocity distribution includes the following steps: (1) the definition of entropy, (2) the principle of maximum entropy, (3) constraint equations, (4) the maximization of the entropy, (5) derivation of probability distribution (6) determination of Lagrange multipliers and finally (7) derivation of the velocity distribution. Each of these steps is discussed in the following.

2.1. Renyi entropy

Renyi [13] defined a generalized form of entropy called Renyi entropy which specializes into the Shannon entropy and others. Renyi [13] expressed the entropy as

$$H_\alpha(p_1, p_2, \ldots, p_n) = \frac{1}{1-\alpha} \log_2 \left(\sum_{i=1}^{n} p_i^\alpha \right) \quad (1)$$

where α is a parameter which is greater than zero and not equal to 1. Here n = number of values that the random variable takes on; p_i's are the probability of the random variable for each $i = 1, 2, \ldots, n$. H_α is maximum when $p_i = \frac{1}{n}$ for each $i = 1, 2, \ldots, n$. From Eq. (1), it can be shown that Renyi entropy reduces to the Shannon entropy as the limiting case $\alpha \to 1$