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Q1 # Pinning synchronization of discrete dynamical networks with delay coupling

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HIGHLIGHTS

- Find out that subsystems can be synchronized when adding pinning control.
- Provide two interesting numerical examples.
- Establish criteria based on Lyapunov stability theory and linear matrix inequalities.

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ABSTRACT

The purpose of this paper is to investigate the pinning synchronization analysis for nonlinear coupled delayed discrete dynamical networks with the identical or nonidentical topological structure. Based on the Lyapunov stability theory, pinning control method and linear matrix inequalities, several adaptive synchronization criteria via two kinds of pinning control method are obtained. Two examples based on Rulkov chaotic system are included to illustrate the effectiveness and verification of theoretical analysis.

1. Introduction

Synchronization is a phenomenon which is of fundamental scientific interest in linear or nonlinear complex dynamical systems. Since 1990 by Pecora and Carroll [1], chaotic synchronization has become a focal research topic. Until now, many control methods have been developed including Pinning adaptive control [2–7], impulsive control [8,9] and intermittent control [10–12] etc. Most of synchronization criteria are concerned with continuous-time dynamical systems with delay or non-delay coupling [2–7,10,11]. But, in todays digital world, discrete-time dynamical systems are widely used because of computer-based simulation and computation in many fields of science and engineering applications, such as image processing, time series analysis, quadratic optimization problems, system identification and discrete analog of continuous systems etc. Naturally, synchronization of discrete-time systems is also an interesting feature. In Refs. [8,9,5] the authors obtained some synchronization criteria for discrete dynamical networks with non-delayed or delayed couplings under impulsive control. In Refs. [13–16] the authors discussed (generalized) synchronization in discrete-time chaotic systems. In Refs. [17,18] synchronization with different complex topological architectures were investigated, they found out that neurons with regular connections have weaker synchronization than those with rand, small-world or scale-free networks. In (Q3) this paper, we propose an example to illustrate that subsystems cannot be synchronized only with linear/nonlinear coupling

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but they can be synchronized when pinning control is added, which also implies that synchronization criteria established for discrete-time chaotic systems in Refs. [13,14,16] cannot be used. By the way theoretical analysis based on the Lyapunov stability theory and linear matrix inequalities are also carried out. This is the main purpose of why we writing this paper.

The rest of the paper is organized as follows: In Section 2, we propose a general discrete complex dynamical network model, and some preliminaries and lemmas are also given. In Section 3, we use the real-time adaptive pinning controllers, obtained some synchronization criteria for the general complex dynamical networks with both identical and nonidentical topology structure. The sufficient condition of synchronization for general complex dynamical networks via the delay adaptive pinning controllers are given in Section 4. Numerical examples are included in Section 5. Finally, we draw our conclusion in Section 6.

2. Preliminaries and mathematical models

Consider a generally controlled complex delayed dynamical system consisting of N nodes, with each node being of m dimensions, the system is described by the following equations:

$$x_i(t+1) = f(x_i(t)) + c_1 \sum_{j=1}^{N} a_{ij} \Gamma g(x_j(t)) + c_2 \sum_{j=1}^{N} b_{ij} \Gamma g(x_j(t-\tau)) + u_i(t) \tag{2.1}$$

where $1 \le i \le N$, $x_i(t) = (x_{i1}(t), \ldots, x_{im}(t))^T \in \mathbb{R}^m$ is the state variable of node i, $f : \mathbb{R}^m \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^m$ are continuously differentiable functions. $\tau > 0$ is the time delay, c_1 and c_2 are two parameters of coupling strengthen, $\Gamma = (\gamma_{ij}) \in \mathbb{R}^{m\times m}$ is an inner-coupling matrix, $u_i(t)$ is an adaptive controller. $A = (a_{ij})_{N\times N}$ and $B = (b_{ij})_{N\times N}$ represents the adjacency configuration of the network with the non-delay and delay couplings. If there is a link from node i to node j, then $a_{ij} = a_{ji} > 0$ $(j \ne i)$; otherwise, $a_{ij} = a_{ji} = 0$. Similarly as B. Such a model can be described as a regular network or a random one by the E–R, small-world [19] or scale-free [20] network respectively. Moreover, $a_{ii} = -\sum_{j\ne i} a_{ij}$ and $b_{ii} = -\sum_{j\ne i} b_{ij}$.

Definition 1. The discrete-time dynamical networks (2.1) is said to achieve asymptotic synchronization if

$$x_1(t) = x_2(t) = \cdots x_N(t) = s(t), \quad \text{as } t \to \infty,$$

where $s(t) \in \mathbb{R}^m$ is a solution of an isolated node, satisfying $s(t+1) = f(s(t))$.

Define error vectors as

$$e_i(t) = x_i(t) - s(t), \quad 1 \le i \le N.$$

To realize the network synchronization, the controllers u_i should guide the error vectors $e_i(t)$ to approach zero as t goes to infinity, as

$$\lim_{k\to\infty} \|e_i(t)\|_2 = 0, \quad 1 \le i \le N.$$

Let us present a lemma for later use.

Lemma 1 ([3]). *Suppose $H = (h_{ij})_{N\times N}$ is a real symmetric and irreducible matrix, in which $h_{ij} \ge 0$ $(i \ne j)$ and $h_{ii} = -\sum_{j\ne i} h_{ij}$, nonzero matrix $D = diag(d_1, d_2, \ldots, d_N)$ satisfies $d_i > 0$ $(1 \le i \le N)$. Let $Q = H - D$, then*

(i) *all the eigenvalues of Q are less than 0;*
(ii) *there exists an orthogonal matrix, $\Phi = (\phi_1, \phi_2, \ldots, \phi_N) \in \mathbb{R}^{N\times N}$, such that*

$$Q^T \phi_k = \lambda_k \phi_k \quad (k = 1, 2, \ldots, N),$$

where $\lambda_1, \lambda_2, \ldots, \lambda_N$ are the eigenvalues of Q.

3. Synchronization of discrete dynamical networks via real-time pinning controllers

Choose the adaptive controllers as follows

$$\begin{cases} u_i(t) = -q\Gamma e_i(t), & 1 \le i \le l, \\ u_i(t) = 0, & \text{otherwise}, \end{cases} \tag{3.1}$$

where the feedback gain $q > 0$. Now we divide our discussion into two cases: Case 1 $(A = B)$ and case 2 $(A \ne B)$.

Q4 **Case 1**. Assume that the matrices A and B are identical i.e., $A = B$. Rewrite the dynamic system (2.1) as the following form:

$$X(t+1) = F(X(t)) + c_1 G(X(t)) \Gamma A^T + c_2 G(X(t-\tau)) \Gamma A^T + U(t), \tag{3.2}$$

where $X(t) = (x_1(t), x_2(t), \ldots, x_N(t)) \in \mathbb{R}^{m\times N}$, $F(X(t)) = (f(x_1(t)), f(x_2(t)), \ldots, f(x_N(t)))$, $G(X(t)) = (g(x_1(t)), g(x_2(t)), \ldots, g(x_N(t)))$, $U(t) = (u_1(t), u_2(t), \ldots, u_N(t))$.

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