Physica A 450 (2016) 531–540

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Fractal analysis of the short time series in a visibility graph method

Ruixue Li [a], Jiang Wang [a], Haitao Yu [a], Bin Deng [a], Xile Wei [a], Yingyuan Chen [b,*]

[a] *School of Electrical Engineering and Automation, Tianjin University, Tianjin, PR China*

[b] *School of Precision Instrument & Opto-electronics Engineering, Tianjin University, Tianjin, PR China*

HIGHLIGHTS

- The visibility graph is applied to analyze short fractal time series.
- A combined method is proposed to estimate Hurst exponent.
- This combined method can provide reliable estimation for short fractal time series.
- The optimal bound of power-law behavior is the first local minimum of K–S statistic.

ARTICLE INFO

Article history:
Received 7 August 2015
Received in revised form 24 November 2015
Available online 13 January 2016

Keywords:
Short time series
fBm
The visibility graph
Hurst exponent
Maximum likelihood estimation
Kolmogorov–Smirnov statistic

ABSTRACT

The aim of this study is to evaluate the performance of the visibility graph (VG) method on short fractal time series. In this paper, the time series of Fractional Brownian motions (fBm), characterized by different Hurst exponent H, are simulated and then mapped into a scale-free visibility graph, of which the degree distributions show the power-law form. The maximum likelihood estimation (MLE) is applied to estimate power-law indexes of degree distribution, and in this progress, the Kolmogorov–Smirnov (KS) statistic is used to test the performance of estimation of power-law index, aiming to avoid the influence of droop head and heavy tail in degree distribution. As a result, we find that the MLE gives an optimal estimation of power-law index when KS statistic reaches its first local minimum. Based on the results from KS statistic, the relationship between the power-law index and the Hurst exponent is reexamined and then amended to meet short time series. Thus, a method combining VG, MLE and KS statistics is proposed to estimate Hurst exponents from short time series. Lastly, this paper also offers an exemplification to verify the effectiveness of the combined method. In addition, the corresponding results show that the VG can provide a reliable estimation of Hurst exponents.

1. Introduction

In mathematics, the fractal has not been exactly defined. Instead, it can be described as a set with subtle, irregular and self-similar structures. In nature, a great number of natural phenomena have fractal features, such as earthquakes, landslides, heartbeat, vegetation patterns, river flow and rainfall [1]. Since Mandelbrot extended the concept of the fractal to time series [2], many researches have indicated that the temporal evolution of many variables are also fractal, such as variables in Physics, technology, biology, geophysics, economics, and psychology [3–8]. These findings suggest that a system can

* Corresponding author. Tel.: +86 22 27402293; fax: +86 22 27402293.
E-mail address: gerard@tju.edu.cn (Y. Chen).

http://dx.doi.org/10.1016/j.physa.2015.12.071

organize its structures with the same method in different timescale, which reveals the importance of fractal to further understand the working mechanism of the system.

In fractal analyses, the power-law form should extend over a range of more than one decade, which is directly related to the sample size. However, fractal methods cannot give reliable results with series shorter than 2^{12} data points, especially devoted to physiological research. The collection of long series in some fields is impossible because of the limitation of experiments or data attributes [9]. And we get into similar trouble in analyses of acupuncture-evoked spike train which cannot be long enough to avoid the instability caused by manual operations. Hence, it is necessary and meaningful to estimate the exact fractal exponents from fractal time series shorter than 2^{12}. So far, some classical methods have been applied into the fractal analysis with such short time series, such as scaled windowed variance method [10], rescaled range analysis and dispersional analysis [11], detrended fluctuation analysis (DFA) [12], maximum likelihood estimation [13] and so on. Then, this paper focuses on the performance of the visibility graph (VG), which can be considered as an alternative method for the fractal analysis with short time series.

In 2008, the VG is proposed by Lacasa et al. based on the graph theory and the visibility rule. The properties of the VG mapping network are inherited from the original time series. The stochastic network is mapped from stochastic time series, regular network from periodic time series and scale-free network from fractional time series [14]. The fractal exponent of series is related to the power-law index of corresponding mapping network. At present, the VG has been used to analyze large-scale series in different fields, including psychology [15], physics [16–18] and economics [19,20]. For example, Lacasa et al. applied the VG to analyze series in Psychology and behavior [15]; Gao et al. used the VG method to the analysis of the gas–liquid two-phase flow [17]; Qian et al. investigated 30 world stock market indices through their visibility graphs by adopting the visibility algorithm to convert each single stock index into one visibility graph [20]. However, the VG fractal analysis for short time series has been nearly unexplored by now. In the application of the VG to short time series, there are two key problems needed to be solved. Firstly, the degree distribution of mapping network, mapped from short fractal time series, cannot obey a power-law distribution exactly, where a platform and a heavy tail occur [21,22]. It inevitably results in a bias of estimation of fractal exponents. Secondly, it will directly affect the accuracy of the estimation of fractal exponents without considering sample size [9].

The present studies were based on the dichotomous model emphasized by Eke et al. [23]. The fBm was defined by Mandelbrot and van Ness in 1968 [24]. By utilizing the scaling law, it is characterized as $\langle \Delta x^2 \rangle \propto \Delta t^{2H}$, where H is Hurst exponent, namely a specific scale factor of fractal time series, satisfying that $0 < H < 1$. As another family of fractal processes, the fractal Gaussian noise (fGn) is defined as the series of successive increments in an fBm [9]. In this study, the short fBm time series are used to verify the effectiveness and reliability of the VG, where a method combining the maximum likelihood estimation (MLE) and Kolmogorov–Smirnov (KS) statistic is proposed to reduce the resulted bias. In addition, the sample size is considered as a parameter in the relationship between fractal exponents and power-law indexes for the improvement of accuracy.

The paper is organized as follows. The VG method, the generation of short fBm time series and an integration of MLE and KS statistic are introduced in Section 2. Section 3 shows simulation results of fractal analysis with short fBm time series and the procedure to estimate Hurst exponents is concluded. Section 4 adopts the proposed scheme to combine VG, MLE and KS statistic to an example of short fBm time series to prove its effectiveness, and at the same time, the DFA is applied to the same data. Finally, the conclusion and discussion are given in the last section.

2. Methods

2.1. Generation of fBm time series

Based on the method identical to Delignieres et al. [9], this paper generates 63 data sets of the fBm time series, each of which is determined by two parameters, namely the length of series N ($N = 2^5, 2^6, \ldots, 2^{12}$) and the Hurst exponent H ($H = 0.1, 0.2, \ldots, 0.9$). Each data set contains 40 fBm series with the same N and H to ensure the statistical accuracy and reliability of the results. Fig. 1 shows graphical examples of fBm time series with different N and H.

2.2. The visibility graph

The VG, proposed by Lacasa et al., is a tool analyzing time series by mapping a time series into a network [14]. The visibility criteria is established as follows: for two arbitrary points (t_a, y_a) and (t_b, y_b) in the time series, if any other point (t_c, y_c) between them satisfies

$$y_c < y_a + (y_b - y_a)\frac{t_c - t_a}{t_b - t_a} \tag{1}$$

the corresponding two nodes in the mapped graph are connected. Obviously, those nodes mapped from neighbor points must be connected in graph.

In the light of the criterion (1), the time series of length N is mapped into a network including N nodes. Previous studies indicated that stochastic network is mapped from stochastic time series, regular network from periodic time series and