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A method of characterizing network topology based on the breadth-first search tree

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HIGHLIGHTS

- A method is proposed to deeply characterize network topology.
- A similarity coefficient is defined to quantitatively distinguish networks.
- The similarity coefficient can quantitatively measure the topology stability of the network generated by a model.
- The network generated by a mode is more and more stable with the increasing of the network scale.
- For a network model, a broader node degree distribution will make the network generated by the model more unstable.

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ABSTRACT

A method based on the breadth-first search tree is proposed in this paper to characterize the hierarchical structure of network. In this method, a similarity coefficient is defined to quantitatively distinguish networks, and quantitatively measure the topology stability of the network generated by a model. The applications of the method are discussed in ER random network, WS small-world network and BA scale-free network. The method will be helpful for deeply describing network topology and provide a starting point for researching the topology similarity and isomorphism of networks.

Generally speaking, network is a set of interconnected nodes, where a node is an element of a natural or man-made system. Network science is an emerging, highly interdisciplinary research area that aims to develop theoretical and practical approaches for understanding the natural and man-made systems. The last decade has witnessed the birth of a new movement of interest and research in the study of complex networks [1]. The study of complex networks is pervading all kinds of sciences today, from physical, biological to social science [2,3], and the network application is also studied [4–7]. Many real complex networks have emerged some common characteristics, such as small world [8], scale-free [9]. Therefore, some important network models with real network characteristics have been proposed. For example, BA scale-free network [9], ER random network [10] and WS small-world network [11].

The development of network science depends on the precise anatomy of network topology. The network topology always affects the function and the behavior of a dynamic system [12]. For example, the topology of social networks affects the spread of information and disease [13], and the topology of the power grid affects the robustness and stability of power transmission [14]. The networks in the ensemble with the same degree distribution could have different connection details, which could lead to different dynamics phenomena. The apparent ubiquity of complex networks leads to a fascinating set

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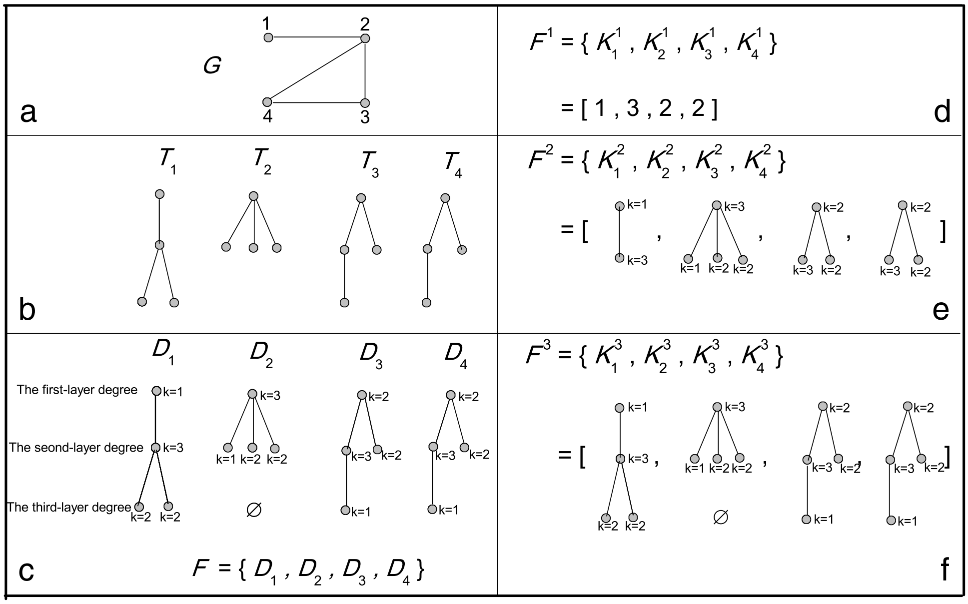

Fig. 1. The schematic illustration of the method. G is a network with four nodes and four edges. The number around a node is the degree of the node in D_1, D_2, D_3 and D_4. D_2 is a two-layer degree-tree, and the third-layer of D_2 is defined null, labeled $\varnothing$. K_2^2, K_3^2, K_3^3 are correspondingly identical to K_2^3, K_4^2, K_4^3, respectively.

of common problems concerning how the network structure facilitates and constraints the network dynamics. Therefore, it is important to characterize the topology of complex networks appropriately.

The research on complex networks begins with the effort of defining concepts and measures to characterize the topology of real networks, such as the degree distributions, degree correlations, average path length, network diameter, clustering coefficient, betweenness and modularity [15,16]. In this paper, a method will be proposed to describe deeply the network topology, and a similarity coefficient is defined to quantitatively distinguish networks and quantitatively measure the topology stability of the network generated by a model. The applications of our method will also be presented on ER random networks, WS small-world networks and BA scale-free networks.

1. Method

In a network, a node can be taken as a root, labeled i. Starting from the root i, a breadth-first search tree can be built, labeled T_i. T_i has a hierarchical structure and contains all the nodes of the network, but the degree of each node cannot be contained in T_i. We make a special provision that the degree of each node in the network is also contained in the breadth-first search tree, and the new breadth-first search tree is called breadth-first search degree-tree(BFSDT), labeled D_i. Therefore, the degrees of all nodes in the network are layered with the breadth-first search tree. The degree-trees of all nodes can be composed to a forest, signed $F = \{D_i, i = 1, \ldots, N\}$, where N is the number of nodes in the network. The forest F can deeply characterize the network topology.

Two concepts of the method are defined as follows:

(i) n-layer degree-tree: The sub-degree-tree which includes the part from the 1th layer to the nth layer in D_i is called n-layer degree-tree, labeled K_i^n, and n is a positive integer. Therefore, K_i^n has a hierarchical structure and is a sub-degree-tree with degrees associated with its nodes.

(ii) Identical n-layer degree-trees: For $\forall K_i^n, K_j^n$, if K_i^n is isomorphic to K_j^n ($K_i^n \cong K_j^n$), and the degrees of any two nodes which meet the relationship of one-to-one isomorphic mapping in K_i^n and K_j^n are equal, then we define K_i^n and K_j^n as the identical n-layer degree-trees.

Because K_i^n is a sub-degree-tree of D_i, the n-layer degree-trees of all degree-trees constitute a set, labeled $F^n = \{K_i^n, i = 1, \ldots, N\}$. F^n can characterize the network topology, and the topology can be characterized better and better with the increasing of n. Specially, for one-layer degree, $n = 1$, $K_i^n = K_i^1$, and $F^n = F^1 = \{K_i^1, i = 1, \ldots, N\}$. One-layer degree-tree K_i^1 is the degree of node i and F^1 is a set of degrees for all nodes. Therefore, general degree is a special case in the method. The schematic illustration of the method is shown in Fig. 1.

For two given networks G_1 and G_2, the n-layer degree-trees of all degree-trees from G_1 constitute a set, labeled F_1^n and the n-layer degree-trees of all degree-trees from G_2 constitute a set, labeled F_2^n. According to the concept of identical n-layer degree-trees, a similarity coefficient can be defined to quantitatively measure the similarity of n-layer degree-trees between