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Rate laws of the self-induced aggregation kinetics of Brownian particles

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HIGHLIGHTS

- Aggregation kinetics is very much sensitive on the nature of multiplicative noise.
- The rate constant as a function of noise strength may have optimum behavior.
- The rate constant as a function of strength of the interference may also have optimum behavior.
- The variation of order parameter and structure factor with noise strength may be non monotonous.
- The variation of order parameter and structure factor with the strength of the interference may also be non monotonous.

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ABSTRACT

In this paper we have studied the self induced aggregation kinetics of Brownian particles in the presence of both multiplicative and additive noises. In addition to the drift due to the self aggregation process, the environment may induce a drift term in the presence of a multiplicative noise. Then there would be an interplay between the two drift terms. It may account qualitatively the appearance of the different laws of aggregation process. At low strength of white multiplicative noise, the cluster number decreases as a Gaussian function of time. If the noise strength becomes appreciably large then the variation of cluster number with time is fitted well by the mono exponentially decaying function of time. For additive noise driven case, the decrease of cluster number can be described by the power law. But in case of multiplicative colored driven process, cluster number decays multi exponentially. However, we have explored how the rate constant (in the mono exponentially cluster number decaying case) depends on strength of interference of the noises and their intensity. We have also explored how the structure factor at long time depends on the strength of the cross correlation (CC) between the additive and the multiplicative noises.

1. Introduction

Study of aggregation dynamics is a subject of key interest in the field of non equilibrium statistical mechanics. This is because of the aggregation of small species that is found to occur in different contexts. Common examples are (a) formation of colony by biological species (b) formation of clouds (c) precipitation from molten state or solution [1,2] (d) clustering of molecules [3] (e) polymerization from monomer [4], aggregation of vacancy [5], etc. For further details we refer Refs. [1,4,6,7]. In the clustering process cooperation among individuals is the key point [7,8]. Therefore several authors have

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tried to establish rigorous and quantitative basis of the emergence of cooperation(CP) among the individuals [9–20]. Here we should mention that the environment has an important role on the cooperative activity [21–25]. *The cooperation mechanism is governed by the dual role of noise* [21–25]: *pushing of particles towards the influence zone of larger cluster and but also taking particles away from clusters. The role of additive noise has been studied in Refs.* [21–24]. *Here cluster formation kinetics follows the power law. Very recently effect of colored multiplicative noise has been investigated in Ref.* [25]. *In this case cluster number decays multi exponentially with time. Then there would be a relevant question. What is the aggregation rate law if it takes place in the presence of both additive and multiplicative noises? To explore answer of this question and related issues we have studied the effect of interplay of additive and multiplicative noises on the aggregation kinetics. At the same time to make the present study more general we have considered interference between the noise processes.* The above question may be very relevant in the case of formation of colony by biological species. Recently, Ref. [25] we have mentioned that weather can be treated as a fluctuating environment to the biological population(BP). Another important quantity which may strongly affect the BP is availability of food. It is in general a fluctuating quantity. Fluctuations in food should depend on the fluctuations of weather. Thus two primary random impacts due to food and weather on biological population may be correlated. We have shown that cross correlation between the fluctuating quantities sometimes may have good impact on cluster formation. Another point to be mentioned here is that the effect of fluctuations may not be same everywhere. To include this in the present we have considered multiplicative noise which introduces non homogeneous diffusion.

In the present study we have derived the Fokker–Planck equation in N dimensional space for one dimensional motion of N individuals. In this equation we have identified that an additional drift term may appear in the presence of the multiplicative noise. At the same time, we have also identified that the cross correlation between the additive and the multiplicative noises may induce both drift and diffusion terms. We have explored their role on the aggregation kinetics. Then there would be an interplay between the drift corresponding to self aggregation process and the environment induced drift. *The interplay may account qualitatively appearance of the different laws of aggregation process. At low strength of white multiplicative noise, the cluster number decreases as a Gaussian function of time. If the noise strength becomes appreciably large then the variation of cluster number with time is fitted well by the exponentially decaying function of time.* The rate constant (RC) for the mono exponential decay law increases regularly with increase of strength of additive noise in the absence of cross correlation between noses. If there is an interference between the noises then a minimum appears in the variation RC as a function of the noise strength. But the rate constant monotonically increases as a function of strength of multiplicative noise both in presence and absence of cross correlation. Our another observation in this context is that for intermediate strength of additive noise, the rate constant passes through a minimum during the change of it as a function of strength of CC between noises. But there is no minimum for a wide range of strength of the multiplicative noise for a given intensity of additive noise. We have also explored at long time how the structure factor(SF) depends on the strength of cross correlation of the noises and their intensity. The SF passes through a maximum with increase of strength of additive noise. But it decreases monotonically in the presence of the interference between additive and multiplicative noises. In the variation of the structure factor as a function of strength of multiplicative noise the maximum does not appear for the intermediate strength of the interference between the noises and the structure factor regularly increases as the multiplicative noise strength grows. Finally, we have observed that if the strength of the multiplicative noise is very high compared to additive noise then the structure factor passes through a maximum as a function of cross correlation strength.

The present study, in another context and with proper changes, can also be used to describe pattern formation and chemot-axis phenomena where diffusion competes with a drift induced by chemical or population gradients [26–30].

The outline of the paper is as follows: In Section 2 we have presented the model which describes the self-induced aggregation of Brownian particles. Results of the present study have been discussed in Section 3. The paper concludes in the Section 4.

2. The model

To study the cluster formation kinetics we have considered a system having N individuals. The dynamical variable of ith individual is represented by x_i. It changes according to a majority rule. x_i implies the reputation score of ith member or the position in a possible chemotaxis description or some other amplitude characterizing the role of individual within the framework of population biology. It is assumed that each individual changes x_i by the following stochastic equation of motion [21–25]

$$\dot{x}_i = v(x_i) + g(x_i)\eta_i(t) + \zeta_i. \tag{1}$$

Here $v(x_i)$ denotes the drift, which is given by the following expression

$$v(x_i) = \lambda_0 \frac{w_+(x_i, t) - w_-(x_i, t)}{w_+(x_i, t) + w_-(x_i, t)}, \tag{2}$$

where $w_\pm$ are defined by

$$w_\pm(x_i) = \sum_j \Theta[\pm(x_j - x_i)] \exp\left(-\alpha|x_j - x_i|\right). \tag{3}$$